HYDRAULICS

OF

RIVERS, WEIRS AND SLUICES

THE DERIVATION OF NEW AND MORE ACCURATE FORMULÆ, FOR DISCHARGE THROUGH RIVERS AND CANALS OBSTRUCTED BY WEIRS, SLUICES, ETC., ACCORDING TO THE PRINCIPLES OF GUSTAV RITTER VON WEX

BY

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TO THE MEMORY OF

GUSTAV RITTER VON WEX

THIS WORK IS
DEDICATED

Hofrat Gustav Ritter von Wex
was aulic counsellor and chief director of the
Danube River regulation and improvement at Vienna;
knight of several imperial orders; and member
of many scientific societies. He was born
1811 and died Sept. 26, 1892,
in Ischl, Austria.

PREFACE

It seems strange that the earnest efforts of so high a technical authority as Hofrat von Wex should have failed to interest hydraulic engineers the world over. A careful search through the leading hydraulic literature, with one exception, did not reveal a single comment regarding either the man or his work. Prof. I. P. Church says in his "Mechanics of Engineering," 1906 ed., foot of page 688, " Herr Ritter von Wex in his Hydrodynamik derives formulæ for weirs, in the establishing of which some rather peculiar views in the mechanics of fluids are advanced."

The unquestioned ability of Hofrat von Wex and his very extensive practical experience along the lines he has treated, place his work in the front rank of technical achievement in the specialty of river hydraulics. His views and theories on this subject, while radically different from those of his time, are most rational and sound, and merit the respect and approval of all practical hydraulic engineers.

With a thorough conviction of the high value of the Wex theories, the author has ventured to place them before his profession in a form which he hopes will prove most practical and acceptable.

The general status of our knowledge respecting hydraulics generally, and particularly the subject of weirs, is very unsatisfactory, to say the least. Hence any progress, if it be real progress, means a radical departure from former conceptions.

It is the rule that new and progressive ideas are received with more or less suspicion, which Professor Church expresses in the word "peculiar." As such ideas become more generally known they receive a more charitable reception and are soon tolerated. When all opposition fails they are crowned by final acceptance. May the present work advance to this final state.

At this age of water power development, this little book should enjoy a hearty welcome. It was prepared with the utmost care, and while theoretical in its nature, it was written for the practical man. Simplicity and clearness were the first requirements, followed by a logical and practical arrangement in the presentation of the subject.

It was not deemed advisable to enlarge the work by the addition of mathematical tables as is usually done. On the contrary, such tables are of little value when dealing with general problems and in the author's opinion the most useful aid to the solution of the formulæ here given is a copy of Barlow's tables of squares, cubes, square roots and cube roots, and Zimmermann's Rechentafel, being a multiplication table of all numbers from 1 to 1000 by all numbers from 1 to 100. These universal tools should occupy a prominent place on every engineer's book shelf, and nothing better can be proposed here as labor saving devices.

Attention is called to the valuable information collected in Appendix A, which constitutes a most complete exposition of all known older formulæ for overfalls. This in itself is the best argument which can be presented in defense of the new formulæ to which this work is devoted.

Appendix B contains the solution of a novel problem in Hydraulics, which, so far as known, has never heretofore been solved in any satisfactory manner. The solution there given is theoretically correct, and probably more accurate than the knowable accuracy of the empiric coefficients would really justify.

In Appendix C, all the new formulæ are arranged in tabulations for ready reference, thus avoiding loss of time in picking out special cases from the text. It is believed that this offers a very attractive summary of the most useful contents of the book.

The author here wishes to acknowledge his indebtedness to Herr Wilhelm Engelmann, of Leipzig, Germany, for many favors extended and advice given prior to undertaking the present work. Also to Professor Gardner S. Williams and Mr. Allen Hazen, members Am. Soc. C. E., for their kind permission to use the

tabulations relating to effect of weir crests as published in their "Hydraulic Tables," pp. 71–75.

In conclusion he wishes to express his obligation and thanks to Mr. Alex. Ilich Wolkowyski, C. E., Ass't Eng'r, Isthmian Canal Commission, for valuable assistance rendered in the preparation of this work, also to Messrs. John Wiley and Sons for the most excellent manner in which they have accomplished the publication.

DAVID A. MOLITOR.

WASHINGTON, D.C., December 5, 1907.

CONTENTS

CHAPTER VI.

CHAPTER VII.

APPENDIX A.

APPENDIX B.

APPENDIX C.

DEFINITIONS OF TERMS USED THROUGHOUT THIS WORK

A = discharge area in square feet.

Q = discharge quantity in cubic feet per second.

v = mean velocity of approach in feet per second.

V = mean velocity of discharge in feet per second.

S = the unit pressure at any point of the discharge area.

S_1 and S_2 are special values of S.

g = acceleration due to gravity, in feet per second = 32.09 at the equator, and 32.26 at the pole, both at sea level.

γ = the weight of a cubic foot of water, usually taken as 62.5 pounds.

n = Dubuat's coefficient = 0.67.

μ = empiric coefficient for discharge into free air.

μ_1 = empiric coefficient for submerged discharge.

T = total depth of approach channel in feet.

T_1 = total depth of discharge channel in feet.

H = depth of flow of approach over crest of weir.

H_1 = depth of flow of discharge over crest of weir.

$H_2 = H - H_1$ = difference between the approach and discharge surfaces.

k = depth of water on upstream side of weir and below weir crest.

B = width of approach channel.

b = width of the discharge section, or length of weir crest.

b' = width of a diversion channel.

d = depth at entrance to a diversion channel.

ψ = angle of inclination of the upstream face of a weir dam with the horizontal.

ϕ = angle of inclination of any wing dam with the side of the channel.

θ = angle of inclination of a diversion channel with the main channel.

s = surface slope = fall divided by length, both in feet.

r = mean hydraulic radius $= \dfrac{A}{w}$.

w = wetted perimeter.

D = original depth of any river, previous to placing any obstruction in its course.

Z = total backwater height, measured above the original or natural surface slope.

L = total backwater distance, measured from the crest of the weir or dam obstruction.

l and z = co-ordinates of backwater surface referred to the original slope as the l-axis and origin in the vertical through the crest of the weir.

C = either Bazin's or Kutter's coefficient for the Chezy formula.

HYDRAULICS OF RIVERS, WEIRS AND SLUICES

INTRODUCTION

In treating a subject of this nature it is very important that the language used should be clear and specific. To accomplish this end a few of the most important terms must be accurately defined so that there may be no doubt as to the particular meaning implied by them. Much confusion is often created by a promiscuous use of technical terms, especially when these have received a variety of definitions by different authors. The following definitions will be adhered to throughout the present work.

Hydrostatic pressure is the pressure exerted by the weight of a column of water and acts with equal intensity in all directions. It implies pressure due to water in a state of rest.

Hydrodynamic pressure is that produced by a stream or jet of water impinging on a surface, and may be less than, equal to or greater than the hydrostatic pressure. It is a function of velocity only. It is always expressed as a static head $v^2/2g$.

Hydraulic pressure is the resultant water pressure on any surface caused by any possible combination of both hydrostatic and hydrodynamic pressures.

Velocity in all of the relations here considered will always be regarded as the mean velocity, being equal for all points of the same section normal to the direction of flow. Hence the quantity of discharge passing a given section A per second of time will be $Q = Av$.

Velocity of approach, v, is the mean velocity at such a section of an approach channel where the effect of an overfall is still too small to produce an acceleration.

Velocity of discharge, V, is the mean velocity at such a section of a discharge channel where the flow is again uniform or normal after having been accelerated by passing over an overfall.

An overfall is a vertical drop in the bottom of a channel and may be complete or incomplete accordingly as it is or is not submerged below the lower water level or lower pool.

A complete overfall weir is a weir the crest of which is above the level of the lower pool.

An incomplete overfall weir, usually called a *submerged weir*, is such a weir the crest of which is below the level of the lower pool.

A sluice weir is a weir the flow over which is partially obstructed by a gate, thus producing a condition of flow resembling that through a lateral orifice in the side of a vessel.

The ends of a weir are usually the side walls of the channel. The end walls will be supposed to be vertical while the front and back of the weir or dam may have various dimensions. The crest of the dam may have a variety of forms which are given special consideration in Chapter VII.

Contraction is a term used to designate the diminution in the flow area just beyond the discharge area when the discharge proceeds into free air and is not confined after leaving the discharge section.

Complete contraction is contraction around the entire periphery of the discharge section.

Partial contraction is contraction on only one, two or three sides of the discharge section.

End contraction is contraction on the two ends of a weir. This occurs only when the side walls of the channel are suddenly ended at the vertical through the weir crest.

All the following formulæ are derived without reference to end contractions or shape of weir crest, and the manner of dealing with these special features is discussed in Chapter VII.

In the several chapters, I to VI, the new formulæ for a great variety of weirs and sluices were derived and these were finally tabulated in usable form in Appendix C. Chapter VII is devoted

to a determination of empiric coefficients for the new formulæ and these results are likewise included in the tabulations of Appendix C.

The inconsistencies and irrational constitution of the older formulæ are discussed at some length in Appendix A, to which special attention is called here as furnishing the real justification for the new formulæ. It was not deemed desirable to interrupt the continuity of the argument by discussing old formulæ at every opportunity, and hence these were collected into an Appendix. In this way the reader will better understand the criticisms after having become familiar with the subject matter of the book.

CHAPTER I.

FUNDAMENTAL EQUATIONS.

Flow through Lateral Orifices.

THE fact that the fundamental equations for flow through lateral orifices have been applied by many hydraulicians, without proper modification, to determine the flow through canals and rivers, contracted by the introduction of weirs or sluices, makes it desirable to review the derivation of these equations, and to show that such application is not justifiable.

The fundamental equations for the flow of water through lateral orifices in the vertical sides of a large reservoir, in which the water is perfectly quiet, and is maintained at a constant level, will be

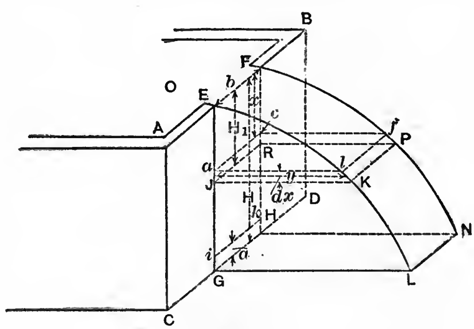


Fig. 1

derived in the following. These equations are generally accepted and will be used in the derivation of the new weir formulæ.

In Fig. 1, $\overline{ABCD}$ represents the vertical wall of the reservoir with the orifice $\overline{EFGH}$, through which the water flows freely into the air. According to the principle of hydrostatics, the pressures at any points, J or G in this orifice, are equal to the columns of water $\overline{EJ}$ and $\overline{EG}$, respectively, and it has been experimentally

established that the velocity with which the water flows through small openings at J or G, is equal to the velocity of a body falling in air through the heights $\overline{EJ}$ and $\overline{EG}$, respectively. Hence, the velocity at the point J is $\overline{JK} = \sqrt{2\,g\overline{EJ}}$, and the velocity at the point G, is $\overline{GL} = \sqrt{2\,g\overline{EG}}$, in which g = acceleration of gravity per second of time.

Using notation indicated in Fig. 1, and calling x the ordinate of a filament of water $\overline{aclf}$ of height dx, and flowing with a velocity y, then the differential equation representing the quantity of water in this filament is

$$dQ = b \,.\, dx \,.\, \sqrt{2\,gx}.$$

To obtain the total quantity of water flowing through the orifice per second, this equation must be integrated between the limits $x = 0$ and $x = H$, giving

$$Q = \int_0^H b dx \sqrt{2\,gx} = b\sqrt{2\,g}\int_0^H x^{\frac{1}{2}}\,dx,$$

or

$$Q = \tfrac{2}{3}\,x^{\frac{3}{2}} b\sqrt{2\,g} + C.$$

For $x = 0$, Q becomes zero, hence the constant C is zero. Therefore, for $x = H$, the quantity of flow through the orifice $b \,.\, H$, per second, becomes

$$Q = \tfrac{2}{3}\,bH^{\frac{3}{2}}\sqrt{2\,g} = \tfrac{2}{3}\,bH\sqrt{2\,gH},$$

in which $b \,.\, H$ equals the area of orifice, and

$$\tfrac{2}{3}\sqrt{2\,gH} = \text{mean velocity}.$$

Since the values of y vary as the square root of the corresponding values of x, the curve $\overline{EKL}$ is a parabola.

However, experiments have shown that the friction existing between the particles of water among themselves and at the edges of the orifice, causes a retardation in this theoretical velocity. Hence, the actual quantity of flow will always be less than the above, and the true equation must contain an empiric coefficient to correct for the combined effect of friction and contraction. The

equation of flow through a rectangular orifice in the vertical side of a reservoir in which the water is perfectly quiet and retained at a constant level may then be written thus:

$$Q = \tfrac{2}{3}\,\mu b H \sqrt{2\,gH} = \tfrac{2}{3}\,\mu b \sqrt{2\,g} H^{\frac{3}{2}} \quad . \quad . \quad . \quad (1)$$

Should the orifice $\overline{EFHG}$ be partly closed by a sluice gate, $\overline{EFRJ}$, then the quantity of flow through the remaining orifice, $\overline{JRHG}$, will be the total quantity as from Eq. (1), less the quantity which is prevented from flowing out by the gate. This quantity is represented by the body $JRHGKPNL$, Fig. 1, and calling the height $\overline{EJ} = H_1$, equation (1) when applied to this case becomes

$$Q = \tfrac{2}{3}\,\mu b \sqrt{2\,g}\,(H^{\frac{3}{2}} - H_1^{\frac{3}{2}}) \quad . \quad . \quad . \quad . \quad (2)$$

Should the sluice gate be lowered to a line $\overline{ik}$, leaving only an orifice of breadth b and height a, which latter is small in comparison to the height H, the water may be assumed to flow through such an orifice under an average pressure $\left(H - \frac{a}{2}\right)$ and the quantity of flow per second will become

$$Q = \tfrac{2}{3}\,\mu ab \sqrt{2\,g\left(H - \frac{a}{2}\right)} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These three fundamental equations have been derived by most hydraulicians, and their correctness having been established by many accurate experiments, they may be generally accepted. However, it must not be forgotten that these equations apply only to cases in which the orifice is very small compared to the size of the reservoir, so that the water in the latter may retain its height and remain unagitated.

Before passing on to the derivation of the weir formulæ, it will be advisable to make an exposition of facts which have been generally neglected by other authors in treating of submerged weirs.

Let Fig. 2 represent the longitudinal section of a river in which is placed a submerged weir $\overline{LM}$ having its crest below the surface $\overline{FG}$ by an amount H_1 and damming the water to a height H

above the crest of the weir. The quantity of water Q, passing over this weir per second, is regarded as being made up of two parts, that flowing into the air through the upper portion $\overline{EK}$ of the entire section $\overline{EL}$, which may be found from Eq. (1); and that flowing

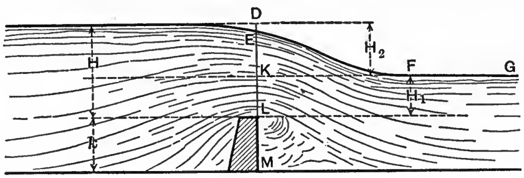


Fig. 2

through the submerged area $\overline{KL}$ under the uniform pressure head $H-H_1$. This is the basis of the argument generally applied in deriving weir formulæ and leads to very erroneous results, as will presently be shown.

Regarding the first of these increments of quantity, the following criticism is offered. As the channel leading up to a weir is always of limited dimensions, and the weir and other possible obstructions, such as wing dams, etc., must affect the conditions of flow through the upper part $\overline{EK}$ of the section $\overline{EL}$, Fig. 2; and since the theoretical Eq. (1), when applied to this flow, does not involve in any way the dimensions of the reservoir, or the channel dimensions in the case of weirs, it follows that the theoretical equation cannot be adapted to the weir condition by the mere introduction of an empiric coefficient. It is also assumed that the velocity of approach exerts an hydraulic pressure only on the area of flow, while it is positively known that this velocity likewise affects the surface of the weir, and all other surfaces of the channel approaching the weir. These pressures are deflected into the area of flow in a manner dependent on the shape of weir and other parts of the channel.

In regard to the second increment of flow, it is assumed that the lower pool exerts a back pressure on the part section $\overline{KL}$, the

same as if the water was not in motion. To prove that this is not in accordance with the existing conditions, the following experiments are cited.

Referring to Fig. 3, in which water flows freely through the irregular vessel $\overline{ABCD}$, and the flow is supplied from a large reservoir, so that the level $\overline{MM_1}$ remains constant, then the resultant hydraulic pressure at any point of the vessel is equal to the hydrostatic pressure at that point, less the velocity height of the water flowing past this same point. (See Ruehlmann, Hydromechanik, pp. 211 and 214.)

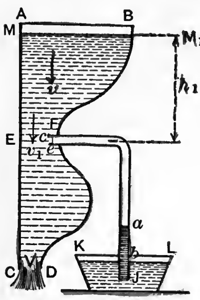


Fig. 3

If in a contracted section $\overline{EF}$, the water flows with a very high velocity v_1, such that the velocity height $\frac{v_1^2}{2g}$ becomes greater than the hydrostatic head h_1 at a certain point F, then the resulting hydraulic pressure on the above hypothesis becomes negative and equal to $\left(h_1 - \frac{v_1^2}{2g}\right)$, indicating a suction. Now, if a glass tube $\overline{FJ}$ be connected with the vessel at F, then the water contained in a vessel $\overline{KL}$ will be drawn up into the tube by an amount $\overline{ab} = \left(\frac{v_1^2}{2g} - h_1\right)$. Also, since the pressure at any section $\overline{EF}$ must be equal in all directions, it is apparent that the same suction would be produced if the opening of the tube were on the lower side at e. Hence, the statement may be made that for the conditions of flow just described, the water flowing past an orifice e in the lower side of a tube $\overline{FJ}$, will produce a suction in this tube equal to

$$ab = \left(\frac{v_1^2}{2g} - h_1\right).$$

To determine the force of impact of water flowing with velocity v through a flume against a disc $\overline{CD}$, Fig. 4 (which question has not yet been satisfactorily solved), Dubuat made numerous experiments and found that the hydraulic pressure on the back face of

the disc $\overline{CD}$ is equal to the hydrostatic head h on the surface, less $0.67 \frac{v^2}{2g}$, proving that in this case there is also a suction on the

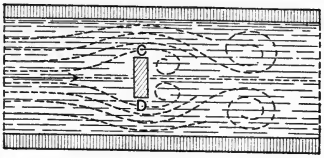


Fig. 4

surface $\overline{CD}$ equal to the velocity height $0.67 \frac{v^2}{2g}$. (See Ruehlmann, Hydromechanik, p. 596.)

The suction in this latter case is, however, less than for the closed vessel, which is supposed to be due to the difference between a closed vessel and an open channel, and also that the eddies produced behind the disc exert a certain impact opposing the suction and thereby diminishing the latter.

Darcy found,* by experimenting with a Pitot tube, that when the orifice was pointed perpendicular to the direction of flow, the hydrostatic column in the tube was lowered by an amount $h_2 = 0.678 \frac{v^2}{2g}$ below the surface of the water, this being the result of suction produced by the water in flowing past the orifice. When the orifice was pointed with the current this suction amounted to only $h_3 = 0.434 \frac{v^2}{2g}$. Since the suction in the latter case should undoubtedly be greater than in the former, it seems reasonable to suppose that in the second case the filaments of water are deflected by the tube in such manner as to diminish the suction effect on the orifice when the tube is pointed with the current.

The fact that the value of the amount of suction found by Darcy on the Pitot tube is less than was obtained for the flow through the vessel in Fig. 3, and less than was found by Dubuat for the disc,

* See Ruehlmann, Hydromechanik, p. 383.

Fig. 4, is probably due to the friction, cohesion and capillarity to be expected by the flow along the small conical pressure tube used.

Until better experiments on these lines shall have become available, the results of Dubuat as $0.67 \frac{v^2}{2g}$ may be safely accepted.

Since the above experiments make it apparent that there is a suction on the lower surface of all incomplete weirs, submerged weirs, and sluice gates, which suction diminishes the hydrostatic counterpressure of the part section $\overline{KL}$, Fig. 2, by an amount $0.67 \frac{v^2}{2g}$, it follows that a larger quantity of flow is permitted through the submerged section, than is assumed under the supposition that the water in the lower pool is perfectly quiet and exerts an hydrostatic pressure on the submerged section over its entire height $\overline{KL}$.

Hence, the generally accepted basis for weir formulæ is erroneous, and in the following chapters new formulæ are derived on the basis of more rational assumptions. Complete overfall weirs are treated first as a matter of convenience.

CHAPTER II.

COMPLETE OVERFALL WEIRS.

A. Derivation of New Formulæ for the Discharge over Complete Overfall Weirs built normally to the Direction of the Current.

THESE formulæ are derived for the following conditions, viz.: That the water reaches the weir section with a certain initial

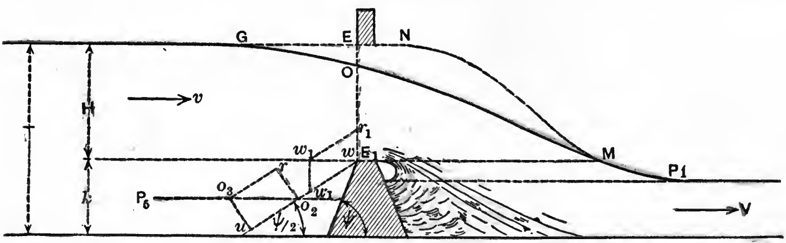


Fig. 5

velocity v; that all the water in the channel must flow over the weir; that the weir is horizontal and has an inclined upstream face;

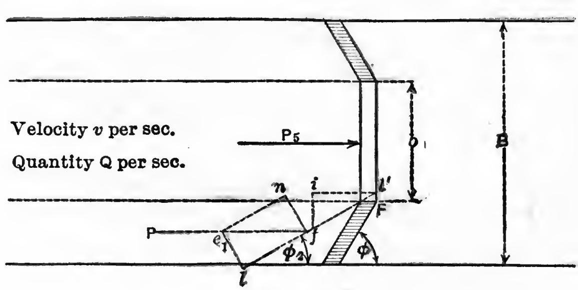


Fig. 6

and that the direction of the weir be normal to the direction of flow. In the general case, a wing dam is assumed located on each side of the weir. (See Figs. 5 and 6.)

Let $g =$ acceleration of gravity.
$\mu =$ coefficient of flow.
$\gamma =$ weight of 1 cubic foot of water.
Other notations as per diagrams.

The breadth of the weir is b; that of the channel is B; the wing walls extend above the water level.

From the dimensions and depth of water

$$Q = B(H + k)v = BTv \quad \ldots \ldots \ldots \quad (4)$$

or

$$v = \frac{Q}{B(H + k)} = \frac{Q}{BT} \quad \ldots \ldots \ldots \quad (5)$$

The projected length of each wing wall on the direction of the weir is $\frac{B - b}{2}$.

The water flowing over the weir takes the surface curve $\overline{GOM}$, but the quantity of flow is the same as if the surface were $\overline{GNM}$, the effective fall being the same in each case.

The forces acting on the entire structure and those acting on the discharge area will now be determined.

1. *The hydrostatic pressure on the section bH is*

Force

$$P_1 = \gamma \frac{bH^2}{2} \quad \ldots \ldots \ldots \quad (6)$$

2. *The hydrodynamic pressure, equally distributed over the area* bH, and resulting from a prism of moving water of area bH and velocity v, is

$$P_2 = \gamma bH \frac{v^2}{2g} = \gamma \frac{bH}{2g}\left(\frac{Q}{B(k + H)}\right)^2 \quad \ldots \quad (7)$$

Force.

3. *The hydrodynamic pressure against a fixed surface* is found to be

$$= \gamma F \frac{v^2}{g} = \gamma q \frac{v}{g}. \quad \ldots \ldots \ldots \quad (8)$$

by Weisbach's experimental law * viz.: "When water flows in a channel enclosed by three sides, the impact against a fixed surface in the channel will be equal to the impact of an isolated stream of water of same cross-section as the water in the channel. As found from the figure, the value of q in Eq. (8) is $v\left(\frac{B-b}{2}\right) H$, and the cross-section of the water in the channel is $\left(\frac{B-b}{2}\right)H=F$. This would indicate that the hydraulic pressure against a fixed surface would be twice that of a stream flowing against an opening of same cross-section.

But since in the above case the water is not confined, and is thus easily deflected towards the area of flow, the quantity q must be assumed for the latter case. Hence the pressure against each wing

$$\text{wall} = P_3 = \gamma q \frac{v}{2g} = \gamma \frac{Hv^2}{2g}\left(\frac{B-b}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which force may be assumed to act in the center of gravity of the obstructed prism.

Regarding the divergence of this water, it should be considered that the filaments passing along the sides of the canal must be deflected through the angle ϕ, while those passing along the line LF are not deflected appreciably; hence, the angle of divergence for the entire prism is taken as $\frac{\phi}{2}$.

The force $P_3 = \overline{e_1 f}$ (see Fig. 6) may be resolved into the components $\overline{lf}$ and nf, the former representing the pressure which is effective against the discharge area, and the latter that expended on the sides of the canal and wing walls.

From Fig. 6,

$$\overline{lf} = \overline{e_1 f} \cos \frac{\phi}{2} = P_3 \cos \frac{\phi}{2} = \gamma q \frac{v}{2g} \cos \frac{\phi}{2}.$$

* Weisbach Experimental Hydraulik, § 423.—Ed. 1855.

The quantity thus reaching the weir cannot continue its flow in this direction and is again deflected into the direction $\overline{il'}$, hence,

$$\overline{il'} = \overline{fl'} \cos \frac{\phi}{2} = \gamma q \frac{v}{2g} \cos^2 \frac{\phi}{2} \quad . \quad . \quad . \quad . \quad (10)$$

which represents the pressure effectively expended against the discharge area, while the force $\overline{if}$ produces a contraction of the flow in the flow area.

Therefore, the total pressure against the discharge area from the quantity of water partially obstructed by the two wing walls is

$$P_4 = 2\,\overline{il'} = \gamma q \frac{v}{g} \cos^2 \frac{\phi}{2} = \gamma H \frac{v^2}{g} \left(\frac{B-b}{2}\right) \cos^2 \frac{\phi}{2}. \quad . \quad (11)$$

This pressure may be represented by a volume of area bH and length t, thus $P_4 = \gamma bHd$ from which

$$t = \frac{P_4}{\gamma bH} = \frac{v^2}{bg} \left(\frac{B-b}{2}\right) \cos^2 \frac{\phi}{2}. \quad . \quad . \quad . \quad (12)$$

4. *The pressure resulting from the volume of water $q_1 = Bkv$ in the lower channel area Bk* is now found.

Since this water q_1 flows in a three-sided channel its hydrodynamic pressure on the sloped weir area may be taken as

$$P_5 = \gamma q_1 \frac{v}{g} = \gamma \frac{v^2}{g} Bk \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which pressure may be regarded as acting in the axis of the channel.

The reasoning previously applied gives for this case an average angle of divergence for this water equal to $\frac{\psi}{2}$.

Resolving P_5 into components $\overline{ro_2}$ and $\overline{uo_2}$, Fig. 5, the latter being effective on the discharge area, it is found from the figure that

$$\overline{uo_2} = \overline{o_3o_2} \cos \frac{\psi}{2} = P_5 \cos \frac{\psi}{2} = \gamma \frac{v^2}{g} Bk \cos \frac{\psi}{2} \quad . \quad . \quad (14)$$

Since this force $\overline{o_2 w}$ tends to lift the overlying water over the weir, it must be deflected horizontally by the counteraction of the upper strata, thus giving the resultant $P_6 = \overline{w_1 w} = \overline{u_1 w} \cos\frac{\psi}{2}$

$$= P_5 \cos^2\frac{\psi}{2}, \text{ or } P_6 = \gamma\frac{v^2}{g} Bk \cos^2\frac{\psi}{2} \quad \ldots\ldots\ldots\ldots \quad (15)$$

This pressure certainly has its maximum effect on the discharge area just at the surface of the weir, which effect gradually diminishes until it becomes zero at the surface of the water. Hence, the total effective result of P_6 on the area bk may be represented by a triangular prism of length b, height H and bottom breadth β,

$$\text{thus:} \quad \gamma\beta b\frac{H}{2} = P_6, \text{ or } \beta = \frac{2\,P_6}{\gamma b H} = \frac{2\,v^2 Bk}{bgH}\cos^2\frac{\psi}{2} \quad \ldots \quad (16)$$

The quantity of water, of depth k, striking the wing walls, can reach the flow area only by material deflection and is here neglected as being small and introducing too many complications.

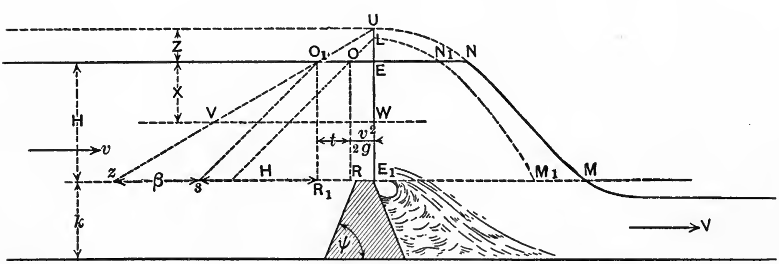


Fig. 7

The individual pressures P_1, P_2, P_4, and P_6 are now combined (See Fig. 7.)

1. The area $\overline{OEE_1R}$ represents $P_2 = bH\,\frac{v^2}{2\,g}$ by making

$$\overline{OE} = \overline{RE_1} = \frac{v^2}{2\,g}.$$

2. Again, by making

$$\overline{OO_1} = \overline{RR_1} = t = \frac{P_4}{\gamma b H} = \frac{v^2}{bg}\left(\frac{B-b}{2}\right)\cos^2\frac{\phi}{2}$$

the area $\overline{OO_1RR_1}$ will represent the pressure P_4 against the discharge area.

3. Also, by making $\overline{O_1R} = \overline{sR_1} = H$, the area of the triangle $\overline{O_1R_1s}$ will represent $P_1 = \mu \dfrac{bH^2}{2}$.

4. Lastly, by making $\overline{sz} = \beta = \dfrac{2\,v^2Bk}{bgH}\cos^2\dfrac{\psi}{2}$ the area $\overline{O_1sz}$ will represent $P_6 = \gamma\beta\dfrac{bH}{2}$.

In conclusion, the sum of the flow areas just designated will make the area $\overline{EE_1zO_1}$, and a prism of this area and the length b will represent the total resultant pressure on the discharge area $\overline{E_1E}$.

Now to compute the velocity from the pressure, continue the line $\overline{zO_1}$ to U, and call y the hydrodynamic pressure at W, of a filament of water distant X, below the surface, and having a velocity V. Also call the surface pressure $\overline{EO_1} = S$, and the pressure at the crest of the weir $= \overline{E_1z} = S_1$.

Then from similarity of triangles $\overline{UEO_1}$ and $\overline{UE_1z}$,

$$\overline{UE} : \overline{EO_1} = \overline{UE_1} : \overline{E_1z},$$

or
$$Z : S = (Z + H) : S_1 \text{ or } Z = \frac{SH}{S_1 - S};$$

also from similarity of triangles $\overline{UWV}$ and $\overline{UE_1z}$

$$\overline{UW} : \overline{VW} = \overline{UE_1} : \overline{E_1z},$$

or
$$(Z + X) : y = (Z + H) : S_1 \text{ or } y = S_1\left(\frac{Z + X}{Z + H}\right).$$

Hence, $V = \mu\sqrt{2\,gy} = \mu\sqrt{2\,gS_1\left(\dfrac{Z + X}{Z + H}\right)}$.

The quantity dQ flowing through an element dx at the point W with velocity V and breadth b will be

$$dQ = bVdx = \mu b dx \sqrt{2\,gS_1\left(\frac{Z+X}{Z+H}\right)} \quad . \quad . \quad . \quad (17)$$

To find Q it is necessary to integrate Eq. (17) between limits $X = 0$ and $X = H$, and obtain

$$Q = \mu b \sqrt{\frac{2\,gS_1}{Z+H}} \int_0^H (Z+X)^{\frac{1}{2}}\,dx = \frac{2}{3}\mu b \sqrt{\frac{2\,gS_1}{Z+H}}(Z+X)^{\frac{3}{2}} + C;$$

when $X = 0$, then $Q = 0$, hence the constant becomes

$$C = -\frac{2}{3}\mu b \sqrt{\frac{2\,gS_1}{Z+H}} \cdot Z^{\frac{3}{2}};$$

and by substituting this value for C in the above expression, and also substituting H for X, the discharge through the area bH, in cubic feet per second, is obtained as follows:

$$Q = \frac{2}{3}\mu b \sqrt{\frac{2\,gS_1}{Z+H}}(Z+H)^{\frac{3}{2}} - \frac{2}{3}\mu b \sqrt{\frac{2\,gS_1}{Z+H}} Z^{\frac{3}{2}}$$

or

$$Q = \frac{2}{3}\mu b \sqrt{\frac{2\,gS_1}{Z+H}}\left[(Z+H)^{\frac{3}{2}} - Z^{\frac{3}{2}}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Now substituting for Z its value $\left(\frac{HS}{S_1 - S}\right)$ and reducing, the following form is obtained:

$$Q = \frac{2}{3}\mu b \sqrt{2\,g\left(\frac{H}{S_1 - S}\right)}\left[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right]. \quad . \quad . \quad . \quad (19)$$

Since $S = \overline{EO_1}$, and $S_1 = \overline{E_1 z}$, these values become, in accordance with previous deductions,

$S = \frac{v^2}{2\,g} + \frac{v^2}{bg}\left(\frac{B-b}{2}\right)\cos^2\frac{\phi}{2}$ from which, by substitution for $v = \frac{Q}{B\,(k+H)}$,

$$\left.\begin{aligned} S &= \frac{1}{2\,g}\left(\frac{Q}{B\,(k+H)}\right)^2\left[1+\left(\frac{B-b}{b}\right)\cos^2\frac{\phi}{2}\right] \\ \text{and}\qquad S_1 &= S + H + \frac{2\,Bk}{bgH}\left(\frac{Q}{B\,(k+H)}\right)^2\cos^2\frac{\psi}{2} \end{aligned}\right\} \quad . \quad . \quad (20)$$

I. *For a straight weir perpendicular to the channel and with vertical face*, Eqs. (19) and (20) become

$$\left.\begin{aligned} S &= \frac{1}{2\,g}\left(\frac{Q}{B\,(k+H)}\right)^2\left[1+\left(\frac{B-b}{2\,b}\right)\right], \\ S_1 &= S + H + \frac{Bk}{bgH}\left(\frac{Q}{B\,(k+H)}\right)^2, \\ Q &= \frac{2}{3}\,\mu b\sqrt{2\,g}\left(\frac{H}{S_1-S}\right)\left[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right]. \end{aligned}\right\} \quad . \quad . \quad . \quad (21)$$

II. *For same weir without wing walls, whence* $B = b$,

$$\left.\begin{aligned} S &= \frac{1}{2\,g}\left(\frac{Q}{b\,(k+H)}\right)^2, \\ S_1 &= S + H + \frac{k}{gH}\left(\frac{Q}{b\,(k+H)}\right)^2, \\ Q &= \frac{2}{3}\,\mu b\sqrt{2\,g}\left(\frac{H}{S_1-S}\right)\left[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right]. \end{aligned}\right\} \quad . \quad . \quad . \quad (22)$$

III. *When* $B = b$; $k = 0$; $\phi = 0°$; *and* $\psi = 0°$.

$$\left.\begin{aligned} S &= \frac{1}{2\,g}\left(\frac{Q}{bH}\right)^2 = \frac{v^2}{2\,g}, \\ S_1 &= S + H = \frac{v^2}{2\,g} + H, \\ Q &= \frac{2}{3}\,\mu b\sqrt{2\,g}\left[\left(H+\frac{v^2}{2\,g}\right)^{\frac{3}{2}} - \left(\frac{v^2}{2\,g}\right)^{\frac{3}{2}}\right], \end{aligned}\right\} \quad . \quad . \quad (23)$$

which is Weisbach's formula for complete overfall weirs in rivers. This coincidence is important as it proves conclusively that Weisbach's formula for complete overfall weirs is applicable only to the case shown in Fig. 8, in which the weir offers absolutely no obstruction to the flow. Such weirs are not built.

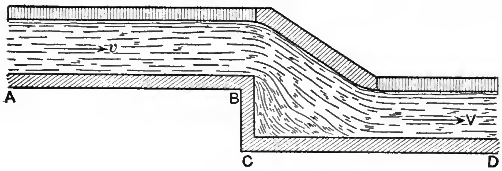


Fig. 8

IV. *When a portion of the flow is diverted through a lateral channel.* In the preceding formulæ it is assumed that all of the approaching water must flow over the weir. Should a portion Q' of this water be wasted through a lateral outlet or over a weir, this quantity must first be found by the formulæ given in the following, and be then subtracted from the entire approaching water Q, for which case Eq. (19) will take the form

$$Q - Q' = \frac{2}{3}\mu b\sqrt{2g}\left(\frac{H}{S_1 - S}\right)[S_1^{\frac{5}{2}} - S^{\frac{5}{2}}]. \quad . \quad . \quad (24)$$

In the equations for S_1 and S, the value of $v = \dfrac{Q}{B(k + H)}$ is true only when the quantity Q arrives immediately in front of the weir with a velocity equal to v.

The hydrodynamic pressures against weir surfaces and wing walls previously found may, however, be modified for the waste water, through a lateral channel or submerged weir, when a definite disposition has been decided upon.

That the above formulæ may be generally applied to complete overfall weirs, of whatever kind, will now be shown.

B. Derivation of New Formulæ for the Discharge over Complete Overfall Weirs, built obliquely to a Channel or Represented in Plan by a Curved or Broken Line.

When it becomes desirable to prevent excessive rise during high water stages, or when, during low water stages, the entire flow is to be utilized through a lateral flume, this may best be accomplished by a diagonal weir of sufficient length.

The case to be treated is shown in Fig. 9, in which $\overline{AADD}$ represents the channel and $\overline{EF}$ the diagonal weir, which is also inclined vertically by an angle ψ with the horizontal. The notation in the previous article will be retained.

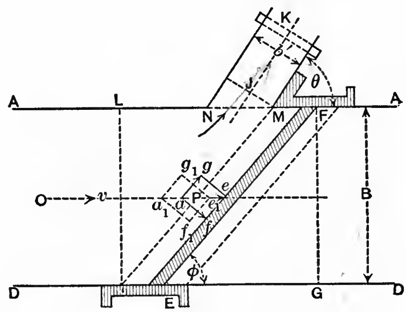


Fig. 9

The water over the weir crest, having a depth H, quantity BHv, and a velocity of approach v exerts an hydrodynamic pressure against the weir section equal to

$$P = \gamma BH \frac{v^2}{2g}.$$

This pressure, being the same in all points of the section, may be considered as acting in the axis Oe and represented in magnitude by the length $ae = P$.

Since P acts on the weir under the angle ϕ, the former may be resolved into components eg and ef, respectively perpendicular

and parallel to the weir. The component $\overline{eg}$ then affects the direct flow over the weir and is found from the following equations;

$$\overline{eg} = \overline{ae} \sin \phi = \gamma B \frac{Hv^2}{2\,g} \sin \phi .$$

If this resultant normal pressure on the weir section be represented by a rectangular prism of length $\overline{EF} = \frac{B}{\sin \phi}$, height H, and thickness t, then $\gamma BH \frac{v^2}{2g} \sin \phi = \gamma \frac{BHt}{\sin \phi}$ from which

$$t = \frac{v^2}{2\,g} \sin^2 \phi = S.$$

This hydraulic pressure t is applied at the surface in front of the weir section and may, therefore, be taken equal to S, as was done in the derivation of Eq. (20).

Taking the section of the weir as in Fig. 7, and considering this section normal to the direction of the weir instead of parallel to the axis of the channel as before, then the rectangle $\overline{EOE_1R}$ (in which now $\overline{EO} = S$) will represent the rectangular prism just mentioned.

Since no wing walls were assumed in Fig. 9, the rectangle $\overline{OO_1RR_1}$ in Fig. 7 becomes superfluous.

The hydrostatic pressure of the advancing water on the weir $\overline{EF}$ is always normal to the weir and is

$$\gamma \overline{EF} \frac{H^2}{2} = \gamma \frac{BH^2}{2 \sin \phi} .$$

This pressure may again be represented by a triangular prism, as was done in the derivation of Eq. (20) by the right angled equilateral triangle O_1R_1s in Fig. 7.

According to the previous derivation of Eq. (20), the total hydrodynamic pressure against the weir between base and crest is $P_5 = \gamma \frac{v^2}{g} Bk$, which pressure may be assumed as acting along the gravity axis $\overline{Oe}$, Fig. 9, and is represented by the line $\overline{a_1e_1}$.

This pressure is again resolved into components $\overline{e_1 f_1}$ and $\overline{e_1 g_1}$ and from the parallelogram of forces is found:

$$\overline{e_1 g_1} = \overline{a_1 e_1} \sin \phi = \gamma \frac{v^2}{g} Bk \sin \phi.$$

This corresponds to the force $\overline{o_2 o_3}$ in Fig. 5.

That portion of the force $\overline{e_1 g_1}$ which acts against the discharge area in the direction $\frac{\psi}{2}$ with the horizontal, is found from the parallelogram of forces and is

$$\overline{o_2 u} = \overline{u_1 w} = \overline{o_2 o_3} \cos \frac{\psi}{2} = \gamma \frac{v^2}{g} Bk \sin \phi \cos \frac{\psi}{2}.$$

In like manner is found the horizontal normal hydrodynamic pressure $\overline{w_1 w}$ from the parallelogram $\overline{r_1 w u_1 w_1}$, Fig. 5, as

$$\overline{w_1 w} = \overline{u_1 w} \cos \frac{\psi}{2} = \gamma \frac{v^2}{g} Bk \sin \phi \cos^2 \frac{\psi}{2}.$$

Since this pressure is a maximum at w (the crest of the weir), and diminishes toward the water's surface, it may be represented by a triangular prism of the length $\frac{B}{\sin \phi}$, height H, and bottom width β found from the equation,

$$\overline{w_1 w} = \gamma \frac{v^2}{g} Bk \sin \phi \cos^2 \frac{\psi}{2} = \gamma \frac{BH\beta}{2 \sin \phi},$$

as
$$\beta = \frac{2\, v^2 k}{gH} \sin^2 \phi \cos^2 \frac{\psi}{2}.$$

If this width β be applied in Fig. 7 as $\overline{sz}$, then the triangle $\overline{O_1 sz}$ will represent a section of the prism of water in question.

Accordingly the total hydrodynamic and hydrostatic pressures acting at the level of the weir crest, will be from Fig. 7,

$$S_1 = \overline{E_1 R} + \overline{R_1 s} + \overline{sz} = S + H + \frac{2\, v^2 k}{gH} \sin^2 \phi \cos^2 \frac{\psi}{2}.$$

Having thus found the hydraulic pressures both at the water's surface and at the crest of the weir, the quantity of flow per second

over a diagonal weir is found by the process adopted in the derivation of Eq. (20) as follows:

$$\left.\begin{aligned} Q &= \frac{2}{3}\mu \frac{B}{\sin\phi}\sqrt{2g}\left(\frac{H}{S_1 - S}\right)\left[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right], \\ S &= \frac{v^2}{2g}\sin^2\phi = \frac{1}{2g}\left(\frac{Q}{B(k+H)}\right)^2 \sin^2\phi, \\ S_1 &= S + H + \frac{2k}{gH}\left(\frac{Q}{B(k+H)}\right)^2 \sin^2\phi\cos^2\frac{\psi}{2}. \end{aligned}\right\} \quad . \quad (25)$$

When it is desirable to construct a weir such that the maximum flow shall not exceed a certain assigned limit, that the weir may become safer and that the flow be more or less deflected away from the sides of the canal and towards its axis; or when it is necessary to supply lateral channels with equal quantities of flow during low and average stages of water, one of the following modifications may be adopted:

1. *The weir consists of two diagonal parts* meeting in the axis of the channel and forming an angle with the apex pointing upstream as in Fig. 10.

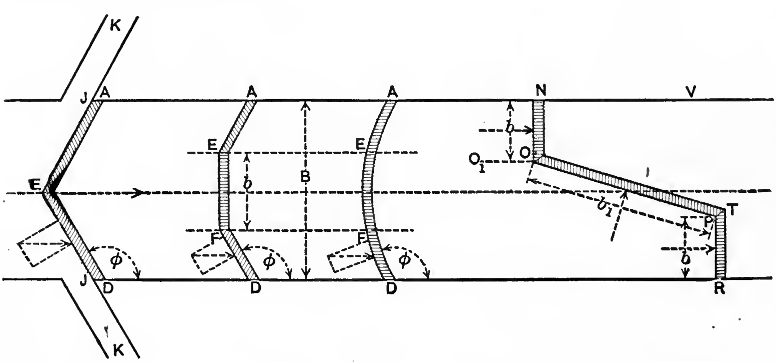


Fig. 10 Fig. 11 Fig. 12 Fig. 13

2. *The weir is like the preceding but the two diagonal sections are separated by a central portion $\overline{EF}$ normal to the direction of flow*, as in Fig. 11.

3. *The weir is curved to the arc of a circle* with convex surface upstream as shown in Fig. 12.

The formulæ for the computation of flow over such weirs as shown in Figs. 10, 11, and 12, may be derived from equations (20) and (25).

When the obliquity of the two halves of the weir, Fig. 10, is the same, Eq. (25) will apply without modification.

For weirs in plan like Figs. 11 and 12, without wing walls, the total flow is found by considering separately the parts AE, FD and EF, in which the curved portion, EF, Fig. 12, is considered straight and normal to the direction of flow.

The flow is thus separated into three parallel filaments, the central one EF exerting a normal *hydraulic* pressure on the central area of flow, found from Eq. (22) as

$$Q_1 = \frac{2}{3} \mu b \sqrt{2g} \left(\frac{H}{S_1' - S'}\right) [S_1'^{\frac{3}{2}} - S'^{\frac{3}{2}}] \quad . \quad . \quad (26a)$$

in which S_1' and S' have the following values:

The pressure of the flow approaching with a velocity v against the weir section is according to the previous derivation $p_2 = \gamma b H \frac{v^2}{2g}$, hence its linear magnitude at the surface per unit of width of weir is

$$S' = \frac{v^2}{2g} = \frac{1}{2g}\left(\frac{Q}{B(k+H)}\right)^2 \quad . \quad . \quad . \quad . \quad (26b)$$

as represented in Fig. 7 by the line $\overline{EO} = \overline{E_1R}$.

The pressure $\overline{OO_1}$, resulting from the wing walls, disappears in the present case because no walls were assumed.

Likewise the normal pressure of the approaching flow against the area of the weir below its crest was found to be

$$P_6 = \gamma \frac{v^2}{g} bk \cos^2 \frac{\psi}{2},$$

by substituting b for B, because in the present case the width b only is considered.

The bottom width of the triangular prism representing this pressure was previously found as

$$\beta = \frac{2\,v^2Bk}{bgH}\cos^2\frac{\psi}{2}$$

which, by substitution of b for B, becomes

$$\beta = \frac{2\,v^2k}{gH}\cos^2\frac{\psi}{2}\,.$$

The magnitude of the total hydraulic pressure of the approaching water at the level of the weir crest is found from Fig. 7 by the equation

$$S_1' = \overline{E_1R} + \overline{R_1s} + \overline{sz} = \frac{1}{2\,g}\left(\frac{Q}{B\,(k+H)}\right)^2 + H$$
$$+ \frac{2\,k}{g\,H}\left(\frac{Q}{B\,(k+H)}\right)^2\cos^2\frac{\psi}{2}\,,$$

or

$$S_1' = S' + H + \frac{2\,k}{gH}\left(\frac{Q}{B\,(k+H)}\right)^2\cos^2\frac{\psi}{2} \quad . \quad . \quad (26c)$$

The quantity of flow per second over the middle section $\overline{EF}$, Figs. 11 and 12, may then be found from Eqs. (26), a, b and c.

The flow over the oblique portions of the weir may now be found from Eq. (25) in the following manner:

Since for an oblique weir the hydraulic pressure of the advancing flow at the surface of the water is equal to S and at the crest of the weir is equal to S_1 from Eqs. (25), in which S and S_1 are found per unit of width of channel, it follows that these equations are also applicable to the oblique weir parts $\overline{AE}$ and $\overline{FD}$, Figs. 11 and 12.

Also, the length of each oblique weir $= \dfrac{B-b}{2\sin\phi}$ and for the two $= \dfrac{B-b}{\sin\phi}$. Then by substituting for $\dfrac{B}{\sin\phi}$ in Eq. (25), the value $\dfrac{B-b}{\sin\phi}$, the quantity of flow over the oblique weir sections is found to be

$$Q_2 = \frac{2}{3}\,\mu\left(\frac{B-b}{\sin\phi}\right)\sqrt{2\,g}\left(\frac{H}{S_1 - S}\right)\left[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right] \quad . \quad . \quad (26d)$$

Hence, the total flow over a weir, polygonal or circular in plan, as in Figs. 11 or 12, is equal to

$$Q = Q_1 + Q_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (26e)$$

When it is required to construct a weir for a medium stage of water such that the high water mark shall not be materially raised, the disposition shown in Fig. 13 has of late been applied, by making the oblique portion $\overline{OP}$ of sufficient length. The section $\overline{NO} = b$ is so chosen as to provide for the partial flow of the mass $\overline{ANO_1O}$ and the sections $\overline{OP}$ and $\overline{PR}$ are so dimensioned as to admit of a regular flow of the quantity $\overline{O_1OUR}$.

By giving the crest of the weir section $\overline{OP}$ a slope equal to the surface slope above the weir, and placing the weir section $\overline{PR}$ level with the lower point P, the approaching flow will reach all parts of the weir with the same initial velocity and constant depth H.

To comply with the conditions of uniform flow, it is necessary to first find the formulæ representing the flow per second over each section of the weir. It is assumed that $\overline{NO} = \overline{PR} = b$, and that the cross-section $\overline{NU}$ is regular.

Since, for the case in hand, $\phi = \psi = 90°$ for the sections $\overline{NO}$ and $\overline{PR}$, the formula (22) will apply to these sections and the flow over each will be represented by the equation

$$q = \frac{2}{3} \mu b \sqrt{2\,g}\left(\frac{H}{S_1 - S}\right)[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}] \quad . \quad . \quad . \qquad (27a)$$

Regarding the flow over the section $\overline{OP}$, it must be considered that since the water flows almost parallel to this section with a velocity v, the hydrostatic pressure H on this portion of the normal section would be diminished by an amount $= \dfrac{0.67\, v^2}{2\,g}$.

But, since the water flowing between cross-sections $\overline{OU}$ and $\overline{PR}$ also flows over the weir $\overline{OP}$ diagonally and hence exerts an hydrodynamic pressure against the discharge area which is somewhat less than $\dfrac{v^2}{2\,g}$ for normal impact, it would seem probable that these

two opposite effects of the advancing water would neutralize each other, and hence it is fair to assume that the advancing flow exerts only an hydrostatic pressure H, and the flow over the section $\overline{OP}$ may be found from the simple formula

$$q_1 = \tfrac{2}{3}\, \mu b_1 H \sqrt{2\, gH} \quad \ldots\ldots \quad (27b)$$

The total flow over the weir $\overline{NOPR}$ will then be

$$Q = 2\, q + q_1 \quad \ldots\ldots\ldots \quad (27c)$$

Such weirs of broken lines offer the advantage that the back water usually caused at times of high water may be reduced to one half that which would result from a weir built normally to a stream, also that the opposite bank $\overline{NU}$ is not so susceptible to washouts as in the case of oblique weirs.

CHAPTER III.

INCOMPLETE OVERFALL WEIRS.

Derivation of New Formulæ for Discharge over Incomplete Overfalls or Submerged Weirs, and through Contracted River Channels.

The same rational suppositions employed in the previous derivations will be followed in the present chapter. Accordingly, the hydrodynamic and hydrostatic pressures, acting on the discharge section, will be ascertained from considerations peculiar to the problem. These pressures will then be graphically combined to produce the resultant pressure areas on the discharge section, and from the latter, analytical formulæ will then be developed.

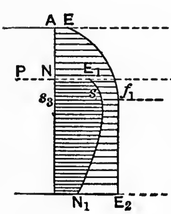


Fig. 14*b*

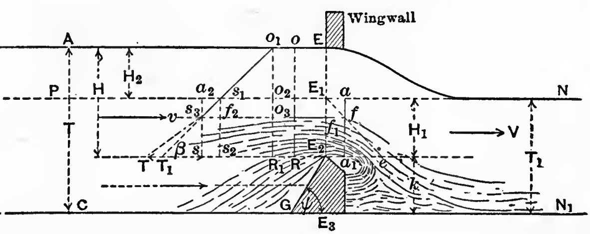


Fig. 14*a*

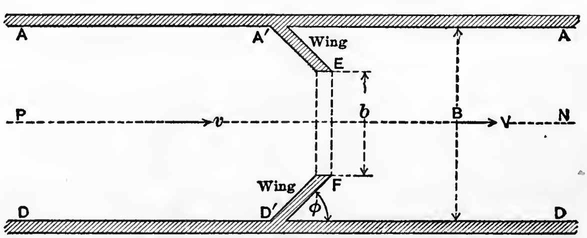


Fig. 14*c*

Fig. 14*a* represents the longitudinal section of a river or canal, obstructed by a submerged weir and wing walls $\overline{A'E}$ and $\overline{D'F}$, shown in plan Fig. 14*c*. All dimensions are indicated on the two figures.

The mean depth of the lower pool above the weir crest is $H_1 = T_1 - k$, when the damming effect of the velocity on the lower level is neglected.

Let Q be the quantity, per second, approaching the weir with a velocity v, which after being discharged over the weir proceeds with a mean velocity V, the amount of which velocity depending on the nature, form and slope of the bed. Also, let μ be the coefficient of flow through the upper part section $\overline{EE_1}$, Fig. 14a, and μ_1 for the lower submerged part E_1E_2.

The flow of approach exerts an hydrodynamic pressure against the discharge area which is uniform over the total depth H and hence may be represented by a rectangular prism $\overline{EOE_2R}$, of height H, length b, and width $\overline{EO} = \overline{E_2R} = \frac{v^2}{2g} = \frac{1}{2g}\left(\frac{Q}{BT}\right)^2$.

The hydrodynamic pressure, against the discharge area, resulting from the flow of approach against the two wing walls and over the height H, is again represented by a rectangular prism $\overline{O_1R_1RO}$ of length b and width $\overline{O_1O} = \overline{R_1R} = \frac{v^2}{bg}\left(\frac{B-b}{2}\right)\cos^2\frac{\phi}{2}$.

The hydrostatic pressure exerted by the head H_2 against the discharge area $\overline{EE_2}$ is represented by the area $\overline{O_1S_1S_2R_1}$, being the resultant of the active hydrostatic pressure $\overline{O_1T_1R_1}$, less the back pressure $\overline{E_1E_2e}$ from the quiescent lower pool. However, the lower water is moving away from the discharge area with a velocity V, thus creating a suction $= 0.67\frac{V^2}{2g} = \frac{nV^2}{2g}$ on the part section $\overline{E_1E_2}$, which will diminish the residual effect of the hydrostatic counterpressure $\overline{E_1E_2e}$ by an amount represented by the rectangle $\overline{E_1E_2a_1a}$. The triangle $\overline{fa_1e}$ thus represents the remaining hydrostatic back pressure on $\overline{f_1E_2}$, while the smaller triangle $\overline{fE_1a} = \overline{s_1a_2s_3}$ represents the residual pressure effect on the discharge area $\overline{E_1f_1}$ due to the suction. Hence, by neglecting this small suction triangle $\overline{fE_1a}$, the net area, $\overline{E_1E_2a_1f} = \overline{s_1s_2ss_3}$, then represents the resulting total suction on the section $\overline{E_1E_2}$. Thus the figure $\overline{EO_1s_3sE_2}$ represents the total effective pressure on the

discharge area $\overline{EE_2}$, except that due to impact against the weir wall $\overline{E_2G}$.

The hydrodynamic pressure against the discharge area resulting from the velocity of approach impinging on the weir wall $\overline{E_2G}$ was found by Eq. (15) as

$$P_6 = \gamma \frac{v^2}{g} Bk \cos^2 \frac{\psi}{2},$$

and is represented by the line $\overline{w_1w}$, Fig. 5. This pressure has a maximum effect on the strata along $\overline{sE_2}$ and may be regarded as entirely dissipated at the level $\overline{f_1s_3}$, hence the triangle sTs_3 of height $\overline{f_1E_2} = \left(T_1 - k - \frac{nV^2}{2\,g}\right)$ and base β may be taken to represent this pressure P_6. The value of β is then found from

$$P_6 = \gamma \frac{\beta b}{2}\left(T_1 - k - \frac{nV^2}{2\,g}\right) = \gamma \frac{v^2}{g} Bk \cos^2 \frac{\psi}{2}$$

and hence, $$\beta = \frac{2\,P_6}{\gamma b\left(T_1 - k - \frac{nV^2}{2\,g}\right)} = \frac{2\,v^2 Bk \cos^2 \frac{\psi}{2}}{bg\left(T_1 - k - \frac{nV^2}{2\,g}\right)}.$$

The area $\overline{EO_1s_3TE_2}$ then represents the total effective pressure on the discharge area, and the unit pressures along certain filaments of the discharge may now be determined.

The pressure along the surface element $\overline{O_1E} = S$ becomes

$$S = \overline{O_1O} + \overline{OE} = \frac{v^2}{2\,g} + \frac{v^2}{bg}\left(\frac{B-b}{2}\right)\cos^2\frac{\phi}{2}$$

$$= \frac{v^2}{2\,g}\left[1 + \left(\frac{B-b}{b}\right)\cos^2\frac{\phi}{2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (28a)$$

The filament $\overline{f_1s_3}$, down to which there will be no counter-pressure from the lower pool, will represent a pressure S_1 evaluated as

$$S_1 = \overline{f_1O_3} + \overline{O_3f_2} + \overline{f_2s_3} = S + H_2 + \frac{nV^2}{2\,g} \quad . \quad (28b)$$

Finally the pressure along the filament $\overline{E_2T}$, at the crest of the weir, is found as

$$S_2 = \overline{E_2 s} + \overline{sT} = S_1 + \beta = S_1 + \frac{2\, v^2 Bk \cos^2 \frac{\psi}{2}}{bg\left(T_1 - k - \frac{nV^2}{2\, g}\right)} \quad (28c)$$

The flow through the part section $\overline{Ef_1}$ proceeds without counter-pressure as for a complete overfall into free air, and by observing that $H_2 + \frac{nV^2}{2\, g} = S_1 - S$, this discharge Q_1 becomes, according to Eq. (19), when $H_2 + \frac{nV^2}{2\, g}$ is substituted for H;

$$Q_1 = \tfrac{2}{3}\, \mu b \sqrt{2\, g}\, [S_1^{\frac{3}{2}} - S^{\frac{3}{2}}] \quad . \quad . \quad . \quad . \quad . \quad (28d)$$

The discharge Q_2 through the lower portion of the section over the height $\overline{f_1E_2} = \left(T_1 - k - \frac{nV^2}{2\, g}\right)$ is found by integration as was done in the derivation of Eq. (19). The velocity along $\overline{f_1s_3}$ will now be $\sqrt{2\, gS_1}$, and that along $\overline{E_2T}$ will be $\sqrt{2\, gS_2}$. Q_2 then becomes

$$Q_2 = \frac{2}{3}\, \mu_1 b \sqrt{2\, g} \left(\frac{T_1 - k - \frac{nV^2}{2\, g}}{S_2 - S_1}\right) (S_2^{\frac{3}{2}} - S_1^{\frac{3}{2}}) \quad . \quad (28e)$$

If the immersed height $\overline{f_1E_2}$ is small, then the mean velocity through this portion of the section may be taken as $\sqrt{2\, g\left(\frac{S_1 + S_2}{2}\right)}$, corresponding to a pressure height $\frac{S_1 + S_2}{2}$, whence

$$Q_2 = \mu_1 b\left(T_1 - k - \frac{nV^2}{2\, g}\right)\sqrt{2\, g\left(\frac{S_1 + S_2}{2}\right)} \quad . \quad . \quad (28f)$$

From these values the total discharge is thus found to be

$$Q = Q_1 + Q_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (28g)$$

Equations (28) may then be regarded as the fundamental equations for incomplete overfalls and submerged weirs. By introducing special values as was done in Eqs. (19) and (20), the following simpler forms are obtained :

When $v = 0$, then

$$\left.\begin{aligned}
S &= 0;\ S_1 = S_2 = H_2 + \frac{nV^2}{2g} \\
Q_1 &= \frac{2}{3}\mu b\left(H_2 + \frac{nV^2}{2g}\right)\sqrt{2g\left(H_2 + \frac{nV^2}{2g}\right)} \\
Q_2 &= \mu_1 b\left(H_1 - \frac{nV^2}{2g}\right)\sqrt{2g\left(H_2 + \frac{nV^2}{2g}\right)} \\
Q &= Q_1 + Q_2 \\
&\text{wherein } H_1 = T_1 - k.
\end{aligned}\right\} \quad . \quad . \quad (29a)$$

For the discharge through a lateral orifice in the vertical wall of a reservoir, into a reservoir of lower level, so as to produce an incomplete overfall, and assuming quiet water in each reservoir, then $\phi = \psi = 90°$ and $v = V = 0$ and calling the height in the lower pool above the bottom of the opening H_1, then Eqs. (28) become

$$S = 0;\ S_1 = S_2 = H_2;$$

$$Q_1 = \tfrac{2}{3}\mu b H_2 \sqrt{2gH_2};$$

$$Q_2 = \mu_1 b H_1 \sqrt{2gH_2} \text{ and finally,}$$

$$Q = Q_1 + Q_2 = \tfrac{2}{3}\mu b H_2 \sqrt{2gH_2} + \mu_1 b H_1 \sqrt{2gH_2} \quad . \quad (29b)$$

This formula coincides perfectly with Dubuat's formula, thus proving that the latter applies only for the special case here assumed.

Figs. 15 and 16*a* represent a river, the flow through which is obstructed by two piers A_1E and D_1F such that the natural surface PN is raised to the level EA. The pressures active on the discharge area for this case will now be determined.

The hydrodynamic pressure, resulting from the velocity of

approach v, is uniformly distributed over the entire depth, and is represented by a rectangle $\overline{EORE_2}$, in which $\overline{EO} = \dfrac{v^2}{2g}$.

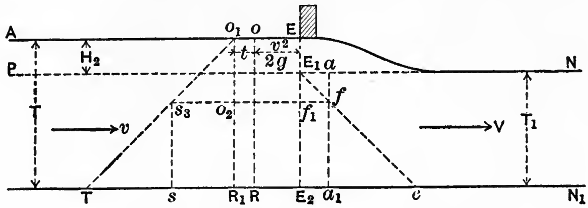


Fig. 15

The hydrodynamic pressure, due to velocity of approach and deflected into the discharge area by the piers, is represented by the rectangle $\overline{OO_1R_1R}$, the width of which is

$$\overline{OO_1} = t = \frac{v^2}{bg}\left(\frac{B-b}{2}\right)\cos^2\frac{\phi}{2}.$$

The suction caused by the velocity of discharge V is represented by the rectangle $\overline{E_1E_2a_1a}$, which subtracted from the hydrostatic pressure due to T_1 and represented by the triangle $\overline{E_1E_2e}$, leaves

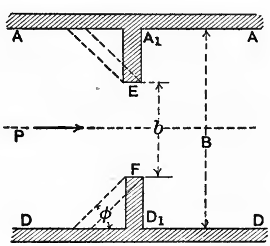


Fig. 16*a*

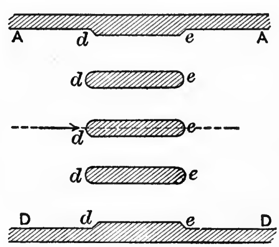


Fig. 16*b*

(after neglecting the small triangle $\overline{aE_1f}$), the net hydrostatic pressure area $fa_1e = s_3Ts$. The total hydrostatic pressure resulting from the head T is given by the triangle $\overline{O_1R_1T}$ from which the net counterpressure s_3Ts is subtracted to leave the effective pressure area $O_1s_3sR_1$.

The total pressure on the discharge area is thus represented by the figure $EO_1s_3sE_2$ from which unit pressures at any point of the discharge section may be obtained.

The discharge, through the part section $\overline{Ef_1}$, takes place as for discharge into open air, while that through the lower portion $\overline{f_1E_2}$ flows with a velocity corresponding to the pressure head $\overline{sE_2}$. Hence, the following pressures are obtained:

$$S = \overline{OE} + \overline{O_1O} = \frac{v^2}{2g} + \frac{v^2}{bg}\left(\frac{B-b}{2}\right)\cos^2\frac{\phi}{2}$$

$$= \frac{v^2}{2g}\left[1 + \left(\frac{B-b}{b}\right)\cos^2\frac{\phi}{2}\right] \quad . \quad . \quad . \quad . \quad (30a)$$

and

$$S_1 = \overline{f_1O_2} + \overline{O_2s_3} = S + H_2 + \frac{nV^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30b)$$

Then, since the whole pressure height $\left(H_2 + \frac{nV^2}{2g}\right) = S_1 - S$, Eq. (19) again furnishes the value for the flow through $\overline{Ef_1}$ as

$$Q_1 = \tfrac{2}{3}\mu b\sqrt{2g}\,[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}]. \quad . \quad . \quad . \quad . \quad (30c)$$

Also, the discharge through the part section $\overline{f_1E_2} = T_1 - \frac{nV^2}{2g}$, subjected to counterpressure and flowing under constant head, is

$$Q_2 = \mu_1 b\left(T_1 - \frac{nV^2}{2g}\right)\sqrt{2gS_1} \quad . \quad . \quad . \quad (30d)$$

Finally, the total flow through the contracted section, of height $T = \overline{EE_2}$, and width $b = \overline{EF}$, Fig. 16*a*, becomes

$$Q = Q_1 + Q_2 = \tfrac{2}{3}\mu b\sqrt{2g}\,[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}] + \mu_1 b\left(T_1 - \frac{nV^2}{2g}\right)\sqrt{2gS_1}. \quad (30e)$$

For the case of a river obstructed by piers, as in Fig. 16*b*, the above formulæ may be employed by substituting the aggregate width of the piers for the width $B - b$ of the former wing walls. This is on the supposition that the piers are sufficiently near to

each other so that their individual damming effects would result in a uniform elevation of the approaching surface.

However, this distinction must be made. In the case of wings, only two contractions occur, while for pier obstructions these contractions are repeated in each opening, thus necessitating special values for the coefficients μ and μ_1 according to the observations of Francis.

In closing this subject a few remarks are here added relative to the effect of obstructions in rivers, and the permanency of river beds.

From many accurate measurements and gagings, it is seen that the velocity is maximum at about one third the depth and diminishes towards the bottom in all cases of unobstructed uniform flow. The discharge curve is then of parabolic form as shown by E_1SN_1, Fig. 14*b*.

On the other hand, for obstructed flow, especially as in Fig. 14*a*, the velocity at the surface is always diminished. It increases gradually with the depth down to the point f_1, where the counter-pressure from the lower pool becomes active, and below this point it remains practically constant. This is illustrated by the discharge curve Ef_1E_2, Fig. 14*b*.

This observation explains the cause for the extensive erosions which always follow the placing of a pier or other obstruction into a river channel.

CHAPTER IV.

SLUICE WEIRS AND SLUICE GATES.

Derivation of New Formulæ for Discharge over Sluice Weirs and through Sluice Gate Openings.

In deriving formulæ for discharge through sluice openings, it is usually supposed that the efflux takes place as for a vessel, Fig. 17, the inside surface of which is assumed to be frictionless and gradu-

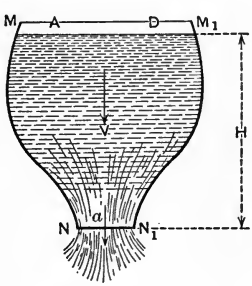


Fig. 17

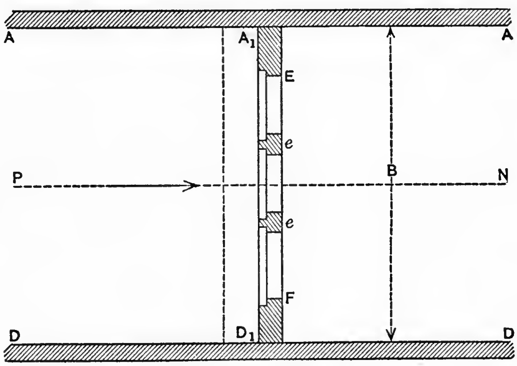


Fig. 18

ally contracted into a conical shape, so that the velocity V, in the enlarged section M_1M, is gradually accelerated toward the opening N_1N. The water in falling, through the height H, acquires a certain *vis viva*, none of which is supposed to be lost in the production of acceleration.*

Now if the flow through such a vessel be compared with that in a river contracted by sluice weirs, Fig. 18, it is almost self-evident that this condition of flow in no wise resembles the above illustration in Fig. 17.

* Ruehlmann, Hydromechanik, pp. 207, 208, 463 and 464.

The water in the river impinging on the wing walls, projections, corners, etc., produces impact. Also, the flow does not arrive at the discharge area with a gradual motion, but rather suddenly, and is then deflected at various angles toward the openings.

All this goes to prove that a considerable portion of the *vis viva* stored in the flow of approach, is necessarily lost by impact and friction.

The older formulæ were also based on the erroneous supposition that the discharge over a submerged weir was resisted by an hydrostatic counterpressure on the discharge area just as for discharge into quiet water, thus disregarding the suction due to velocity of discharge.

Formulæ based on such discordant ideas cannot be expected to furnish correct values for discharge through sluices, etc. For this reason new and more rational formulæ will now be developed with the aid of the principles formerly applied to the derivation of Eqs. (19), (20) and (28).

1. *Formulæ for case illustrated in Fig.* 19, being a sluice weir with submerged discharge.

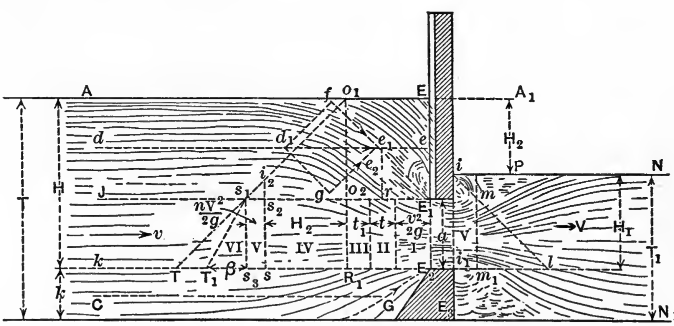


Fig. 19

Figures 18 and 19 represent the plan and longitudinal section, respectively, of a submerged weir A_1D_1, built in a river. The wing walls, A_1E and D_1F, extend above water level and the space EF, Fig. 18, contains three gate openings.

When the gates are raised to a height $\overline{E_1E_2} = a$, the water in the upper pool approaching with a velocity v, will be supposed to exert a pressure head H_2, causing a velocity of discharge V, in the lower pool.

The quantity of discharge Q, passing through the three openings, as above illustrated, will now be found.

The hydrodynamic and hydrostatic pressures, exerted on the discharge area, are separately determined for each of the three following prisms of flow:

1. The lower prism $\overline{CGKE_2}$, of height k.
2. The middle prism $\overline{KE_2E_1J}$, of height a.
3. The upper prism $\overline{JE_1EA}$, of height

$$H - a = T_1 + H_2 - (k + a).$$

Making the sum of the widths of the three gates equal to b, then the total width of the interposed obstruction, including piers and wing walls, will be $B - b$.

The hydrodynamic pressure against each of the orifices may now be represented by a rectangular prism of height a, length $B - b$, and width $\frac{v^2}{2g}$, shown in Fig. 19, as pressure area I.

The hydrodynamic pressure deflected by the wing walls and piers into the discharge area, as found in the derivation of Eqs. (19) and (20), may be represented by a rectangular prism II, of width

$$t = \frac{v^2}{2g}\left(\frac{B-b}{2}\right)\cos^2\frac{\phi}{2}.$$

However, the wing walls in the present case being perpendicular to the canal axis, $\phi = 90°$ and hence,

$$t = \frac{v^2}{2g}\left(\frac{B-b}{4}\right) \quad \text{. . . . .} \quad (31)$$

The upper prism of flow, of depth $H - a$, exerts an hydrodynamic pressure against the three gates and the upper part of the wing walls, active along the gravity axis de and over an area

$$= B[T_1 + H_2 - (k + a)].$$

This pressure is then

$$p = \gamma B[T_1 + H_2 - (k + a)] \frac{v^2}{2g}. \quad . \quad . \quad . \quad (32)$$

Since the lower strata $\overline{JE_1}$ of this prism undergo scarcely any deflection, while the upper strata along $\overline{AE}$ must be deflected 90°, the average deflection for the prism may be taken as 45°. This is represented in Fig. 19, by the direction E_1f.

If the pressure p be represented as a linear magnitude $\overline{d_1e_1}$, decomposed into e_1f and e_1g, then,

$$\overline{e_1f} = \overline{d_1e_1} \cos 45° = \gamma B[T_1 + H_2 - (k + a)] \frac{v^2}{2g} \cos 45°.$$

But $\overline{e_1f} = \overline{E_1e_2}$, may be again resolved into components $\overline{E_1e}$ and $\overline{E_1r}$, from which the required horizontal component is found as

$$\overline{E_1r} = \overline{E_1e_2} \cos 45° = \gamma B[T_1 + H_2 - (k + a)] \frac{v^2}{2g} \quad . \quad . \quad (33)$$

This component exerts a uniform pressure on the discharge area and its effect is represented by a rectangular prism of length b, height a, and width t_1, which latter may be found from Eq. (33) by making $\overline{E_1r} = \gamma abt_1$, whence

$$t_1 = \frac{Bv^2}{4\,abg}\left[T_1 + H_2 - (k + a)\right] \quad . \quad . \quad . \quad (34)$$

which determines the pressure area III, Fig. 19.

The total hydrostatic pressure of the upper pool on the whole discharge area may be represented by the weight of a triangular prism o_1R_1T. The counterpressure from the lower pool is then given by the triangular prism $ii_1l = i_2sT$, which, subtracted from

the former, gives the net effective hydrostatic pressure, represented by the area $o_1 i_2 R_1 s$.

The portion $o_1 o_2 s_2 i_2$, of this latter area, is taken up by the upper portion of the sluice gate, leaving only the effective pressure area $o_2 R_1 s s_2 = \gamma b H_2 a$, shown in Fig. 19, as area IV of width $R_1 s = H_2$.

The discharge velocity V, in the lower pool, produces a suction $\frac{nV^2}{2g}$, represented by the pressure area V, of width $\frac{nV^2}{2g}$.

The triangular prism VI, with base $\overline{T_1 s_3} = \beta$, represents that part of the hydrostatic pressure on the weir $E_2 G$, which, after being twice deflected, finally becomes effective on the discharge area. In the derivation of Eqs. (19) and (20), the value of β was found as

$$\beta = \frac{2\, v^2 B k}{abg} \cos^2 \frac{\psi}{2}.$$

The total *hydraulic* pressure of the upper pool on the discharge area, is then obtained from the summation of all the pressure areas above found. This gives the trapezoidal figure

$$\overline{E_1 s_1 T_1 E_2} = \mathrm{I} + \mathrm{II} + \mathrm{III} + \mathrm{IV} + \mathrm{V} + \mathrm{VI} \quad . \quad . \quad (35)$$

from which the unit pressure on any point of the discharge section may be ascertained.

Thus, the pressure in the upper filament $\overline{s_1 E_1} = S$ is made up of the combined widths of the several prisms I to V, and is evaluated as

$$S = \frac{v^2}{2g} + \frac{v^2}{4bg}(B - b) + \frac{Bv^2}{4abg}\Big[T_1 + H_2 - (k + a)\Big] + H_2 + \frac{nV^2}{2g}$$

or

$$S = \frac{v^2}{2g}\left[1 + \frac{B - b}{2b} + \frac{B}{2ab}\Big(T_1 + H_2 - (k + a)\Big)\right] + H_2 + \frac{nV^2}{2g} \quad (36a)$$

The unit pressure along the filament $\overline{T_1 E_2} = S_1$ will then be

$$S_1 = S + \frac{2\, v^2 B k}{abg} \cos^2 \frac{\psi}{2} \quad . \quad . \quad . \quad . \quad . \quad (36b)$$

By a process similar to that employed in deriving Eq. (19), the quantity of discharge is found to be

$$Q = \tfrac{2}{3}\mu_1 ab\sqrt{2\,g}\left[\frac{S_1^{\frac{3}{2}} - S^{\frac{3}{2}}}{S_1 - S}\right]. \quad . \quad . \quad . \quad . \qquad (36c)$$

When the sluice sill is level with the river bed, making $k = o$, and thus $S_1 = S$, then the flow through a proceeds with the uniform velocity $\sqrt{2\,gS}$, and the discharge may be found from the following simple equation :

$$Q = \mu_1 ab\sqrt{2\,gS} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (36d)$$

2. *Formulæ for the case illustrated in Fig.* 20, for $k = o$ and $T_1 < a$, will now be derived.

The rectangles I, II, and III represent, as in Fig. 19, the cross sections of the pressure prisms effective on the discharge area and resulting from the hydrodynamic pressure due to velocity of

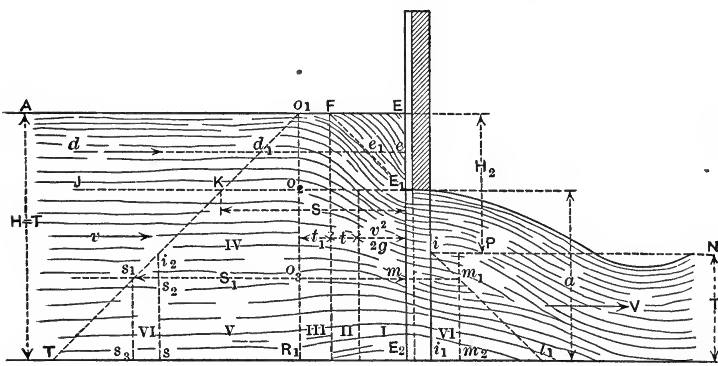


Fig. 20

approach impinging on the discharge area and on the wing walls and upper portion of the gate. Hence, the expressions just found will apply to the present case when $k = o$ is introduced.

Considering the hydrostatic counterpressure of the lower pool PN, active up to a line s_1m_1, then the effective pressure on the discharge area will be given by the triangle o_1R_1T less the triangle o_1o_2K less the triangle i_2sT, leaving the net area $o_2R_1si_2$, which is composed of areas IV and V, Fig. 20.

The area VI finally represents the suction on the discharge section produced by the discharge velocity V, and this suction being taken as uniformly distributed, its horizontal intensity becomes $\overline{s_1 s_2} = \frac{nV^2}{2g}$.

Fig. 20 then furnishes the unit pressures along any horizontal filament.

From this figure also, the following dimensions are found:

$$\left.\begin{aligned}
\overline{E_1E} &= \overline{o_1o_2} = \overline{o_2K} = H_2 + T_1 - a \\
\overline{E_1m} &= a - \left(T_1 - \frac{nV^2}{2g}\right) = a + \frac{nV^2}{2g} - T_1 \\
\overline{o_3s_1} &= \overline{o_1o_3} = H_2 + \frac{nV^2}{2g} \\
\overline{E_2m} &= T_1 - \frac{nV^2}{2g}
\end{aligned}\right\} \quad . \quad (37)$$

The discharge is evaluated in two partial quantities, Q_1 for the flow through the upper part section E_1m, which takes place as for discharge into free air, and Q_2 for the flow through the lower part section E_2m, which is submerged.

Accordingly the resultant hydraulic pressure along the filament $\overline{E_1K} = S$, is found as

$$S = \frac{v^2}{2g} + \frac{v^2}{4bg}(B-b) + \frac{Bv^2}{4abg}[H_2 + T_1 - a] + [H_2 + T_1 - a]$$

or

$$S = \frac{v^2}{2g}\left[1 + \frac{B-b}{2b}\right] + (H_2 + T_1 - a)\left[\frac{Bv^2}{4abg} + 1\right]. \quad . \quad (38a)$$

The total pressure in the lower filament $\overline{s_1m}$, is equal to $S + \overline{E_1m} = S_1$, or

$$S_1 = S + a - \left(T_1 - \frac{nV^2}{2g}\right) = S + a + \frac{nV^2}{2g} - T_1 \quad . \quad . \quad . \quad . \quad (38b)$$

The discharge Q_1, according to the fundamental Eq. (19), now becomes

$$Q_1 = \tfrac{2}{3}\,\mu b \sqrt{2\,g} \left[\frac{a + \dfrac{n\,V^2}{2\,g} - T_1}{S_1 - S} \right] (S_1^{\frac{3}{2}} - S^{\frac{3}{2}})$$

$$= \tfrac{2}{3}\,\mu b \sqrt{2\,g}\,(S_1^{\frac{3}{2}} - S^{\frac{3}{2}})\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (38c)$$

The resultant hydraulic pressure over the lower part section $\overline{E_2 m}$ is uniformly distributed over this depth and has the unit intensity S_1, corresponding to a velocity $\sqrt{2\,gS_1}$, from which Q_2 becomes

$$Q_2 = {}_1 b \left(T_1 - \frac{nV^2}{2\,g} \right) \sqrt{2\,gS_1}\ . \quad . \quad . \quad . \qquad (38d)$$

The total discharge through the orifice of height a is then

$$Q = Q_1 + Q_2\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (38e)$$

3. *Another case of discharge through sluices is illustrated in* Fig. 21. Here the hydraulic pressure is supposed to be very great,

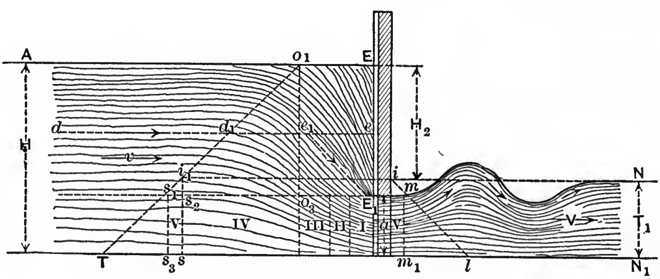


Fig. 21

and the discharge area very small. The velocity at the orifice is greater than the discharge velocity and, in consequence of this retardation, an impact is produced which is expended in elevating the discharge surface into a wave just in front of the sluice opening.

Regarding this phenomena and its effect on the discharge, little reliable information exists. The following solution is offered.

In the previous case all the factors bearing on the discharge were

carefully considered and these apply equally in the present problem with the exception that the wave, when it exists, enters as a disturbing element in determining the particular value of T_1, which is likely to govern the discharge. The matter then resolves itself into finding such a suitable value for T_1.

Obviously, the whole height of the wave cannot be taken as the level of the lower pool, because the wave is merely the result of impact of discharge, and because the wave-crest is immediately followed by a deep wave-hollow.

Also, the velocity at different points of the wave-crest varies in magnitude and direction, and no particular value could be selected as *the* discharge velocity.

When the normal flow is once established, the depth and velocity in the lower pool undoubtedly exert a marked influence on the production and shape of the wave, which effect is probably transmitted back to the discharge area.

Slight changes in the conditions of flow, constantly occurring, may cause the wave to disappear entirely, thus reducing the surface of the lower pool to a level.

It would thus seem most rational to disregard the wave and assume as the most probable lower pool level, the one resulting from normal flow.

However, should the length of the flume be insufficient to develop a continuity of flow through the flume, then the mean height of the wave may be taken as the resisting depth of the lower pool, and the mean velocity resulting from this depth may then be used in the above formulæ.

The values of T_1 and V, in the formulæ of the present chapter, are supposed to be known from gaugings or otherwise. However, when they are not known, as in the case of proposed sluices, then the methods suggested in Chapter VI must be employed.

In the previous formulæ, a certain velocity of approach v was included. When discharge takes place from a lake or other quiescent water, then $v = 0$, and the various pressure areas I, II, and III disappear, thus greatly simplifying the formulæ.

This special condition when introduced into the above equations furnishes the following formulæ for discharge from quiescent water into a flume or canal.

a. For a sluice gate at the inlet to a canal supplied from a lake, case Fig. 19, then $v = 0$ and Eqs. (36) give

$$\left.\begin{aligned} S &= S_1 = H_2 + \frac{nV^2}{2\,g}, \\ Q &= \mu_1 ab \sqrt{2\,g\left(H_2 + \frac{nV^2}{2\,g}\right)} = \mu_1 ab\sqrt{2\,gS}. \end{aligned}\right\} \quad . \quad . \quad (39)$$

b. For a sluice gate at the inlet to a canal supplied from a lake, case Fig. 20, then $v = 0$, and Eqs. (38) become

$$\left.\begin{aligned} S &= H_2 + T_1 - a, \\ S_1 &= H_2 + \frac{nV^2}{2\,g}, \\ Q_1 &= \tfrac{2}{3}\,\mu b\sqrt{2\,g}\,[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}], \\ Q_2 &= \mu_1 b\left(T_1 - \frac{nV^2}{2\,g}\right)\sqrt{2\,gS_1}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (40)$$

c. Should the water be discharged from a high reservoir into one of lower level, by means of a sluice gate, both reservoirs being quiescent and the lower level is above the bottom of the sluice opening, then the function $\frac{nV^2}{2\,g} = 0$, and the hydrostatic counter-pressure from the lower reservoir then becomes effective.

When the discharge of a river is known, then the dimensions of weirs, sluices, etc., as well as backwater height and distance, can be determined by a method of approximation to be discussed in Chapter VI, with the aid of the above formulæ.

By the application of the principles here employed, in the derivation of new formulæ in Chapters II, III, and IV, it will be possible to solve any similar problems by developing formulæ appropriate to such cases.

The formulæ of the present chapter are also applicable to small regulating gates.

CHAPTER V.

BACKWATER CONDITIONS.

Discussion and Formulæ for Backwater Height and Distance.

THE formulæ in the previous chapters deal with problems of obstructed flow due to objects of assigned dimensions, and the backwater height enters as a given function. In the following, the reverse conditions will be treated under three cases.

1. When the dimensions of an existing or proposed weir or sluice are given together with the flow of approach, to find the backwater height.

2. The flow of approach and backwater height being given to find the dimensions of the weir or sluice such that the flow will proceed with this assigned height.

3. Given the backwater height to find the backwater distance and surface curve.

When the flow of approach Q, is given, as for problems under 1 and 2, then one of the unknowns H_2, k or b, can be found for assigned values of the others, by using one of the above formulæ. When these formulæ become too complicated for direct solution of an independent variable, then the method of approximation, by substitution of assumed values for this variable, must be resorted to.

Problems coming under the third head may be solved by the somewhat complicated formulæ given by Professor Ruehlmann, which in the absence of a better solution are here reproduced.

Drift and sedimentation always enter as a disturbing element in river hydraulics, so that no formulæ, however accurate, could be made to permanently satisfy all these changeable conditions.

Professor Ruehlmann gives tabulated values for the ready solution of his formulæ, and the results so obtained agree very well with those of Hagen, Weisbach and Heinemann.

Let D = the original uniform depth of the river.
s = the original natural slope of the river.
Z = the total backwater height measured above the natural surface slope.
L = the total backwater distance measured from the crest of the weir.
z and l are coördinates of the backwater curve, referred to a point O on the natural slope line, vertically above the crest of the weir. (See Fig. 22.)

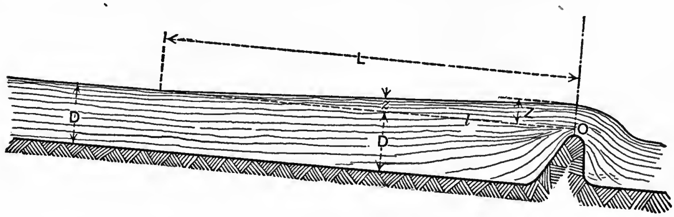


Fig. 22

$f\left(\frac{Z}{D}\right)$ and $f\left(\frac{z}{D}\right)$ are Ruehlmann's functions of $\frac{Z}{D}$ and $\frac{z}{D}$ respectively, the values for which are given in Table I.

Ruehlmann's formula is

$$l = \frac{D}{s}\left[f\left(\frac{Z}{D}\right) - f\left(\frac{z}{D}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad (41)$$

When D, s, Z and z are given, the problem is thus solved. The different values of $\left(\frac{Z}{D}\right)$ and $\left(\frac{z}{D}\right)$ are found from one of the columns 1, of Table I, and opposite these in column 2, are the corresponding functions $f\left(\frac{Z}{D}\right)$ and $f\left(\frac{z}{D}\right)$. These, when substituted in Eq. (41), give the abscissa l corresponding to the ordinate z, for any given D and s.

When D, s, Z and l are given, the value of z is found from Eq. (41), by transformation, thus

$$f\left(\frac{z}{D}\right) = f\left(\frac{Z}{D}\right) - \frac{sl}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Then having found $f\left(\frac{z}{D}\right)$ from Eq. (42), the corresponding value of $\left(\frac{z}{D}\right)$ is given by Table I finally $z = \frac{z}{D} \cdot D$.

In the same manner Z may be found from the following Eq. (43):

$$f\left(\frac{Z}{D}\right) = \frac{sl}{D} + f\left(\frac{z}{D}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

For the total backwater distance L the three quantities z, $\left(\frac{z}{D}\right)$ and $f\left(\frac{z}{D}\right)$ are each zero. The Eq. (41) then becomes:

$$L = \frac{D}{s} f\left(\frac{Z}{D}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

When Table I does not include exactly the values for any given $\left(\frac{Z}{D}\right)$ or $\left(\frac{z}{D}\right)$, then $f\left(\frac{Z}{D}\right)$ or $f\left(\frac{z}{D}\right)$, corresponding to the exact given value, may be found from the following interpolation formula

$$f\left(\frac{z}{D}\right) \text{ or } f\left(\frac{Z}{D}\right) = y = y_1 + (y_2 - y_1)\left[\frac{x - x_1}{x_2 - x_1}\right] \quad . \quad (45)$$

in which

x = the given value of $\left(\frac{Z}{D}\right)$ or $\left(\frac{z}{D}\right)$

x_1 = the next smaller value of x or $\left(\frac{z}{D}\right)$ in Table I.

x_2 = the next larger value of x or $\left(\frac{z}{D}\right)$ in Table I,

y = the required function $f\left(\frac{Z}{D}\right)$.

y_1 = the value $f\left(\frac{Z}{D}\right)$ corresponding to x_1.

y_2 = the value $f\left(\frac{Z}{D}\right)$ corresponding to x_2.

TABLE I.

PROFESSOR RUEHLMANN'S BACKWATER FUNCTIONS FROM HYDROMECHANIK, PAGE 484.

1 $\frac{Z}{D}$ $\frac{z}{D}$	2 $f\left(\frac{Z}{D}\right)$ $f\left(\frac{z}{D}\right)$	1 $\frac{Z}{D}$ $\frac{z}{D}$	2 $f\left(\frac{Z}{D}\right)$ $f\left(\frac{z}{D}\right)$	1 $\frac{Z}{D}$ $\frac{z}{D}$	2 $f\left(\frac{Z}{D}\right)$ $f\left(\frac{z}{D}\right)$	1 $\frac{Z}{D}$ $\frac{z}{D}$	2 $f\left(\frac{Z}{D}\right)$ $f\left(\frac{z}{D}\right)$	1 $\frac{Z}{D}$ $\frac{z}{D}$	2 $f\left(\frac{Z}{D}\right)$ $f\left(\frac{z}{D}\right)$
0.010	0.0067	0.235	1.2148	0.460	1.6032	0.685	1.9077	0.910	2.1800
0.015	0.1452	0.240	1.2254	0.465	1.6106	0.690	1.9140	0.915	2.1858
0.020	0.2444	0.245	1.2358	0.470	1.6179	0.695	1.9203	0.920	2.1916
0.025	0.3222	0.250	1.2461	0.475	1.6252	0.700	1.9266	0.925	2.1974
0.030	0.3863	0.255	1.2563	0.480	1.6324	0.705	1.9329	0.930	2.2032
0.035	0.4411	0.260	1.2664	0.485	1.6396	0.710	1.9392	0.935	2.2090
0.040	0.4889	0.265	1.2763	0.490	1.6468	0.715	1.9455	0.940	2.2148
0.045	0.5316	0.270	1.2861	0.495	1.6540	0.720	1.9517	0.945	2.2206
0.050	0.5701	0.275	1.2958	0.500	1.6611	0.725	1.9579	0.950	2.2264
0.055	0.6053	0.280	1.3054	0.505	1.6682	0.730	1.9641	0.955	2.2322
0.060	0.6376	0.285	1.3149	0.510	1.6753	0.735	1.9703	0.960	2.2380
0.065	0.6677	0.290	1.3243	0.515	1.6823	0.740	1.9765	0.965	2.2438
0.070	0.6958	0.295	1.3336	0.520	1.6893	0.745	1.9827	0.970	2.2496
0.075	0.7222	0.300	1.3428	0.525	1.6963	0.750	1.9888	0.975	2.2554
0.080	0.7482	0.305	1.3519	0.530	1.7032	0.755	1.9949	0.980	2.2611
0.085	0.7708	0.310	1.3610	0.535	1.7101	0.760	2.0010	0.985	2.2668
0.090	0.7933	0.315	1.3700	0.540	1.7170	0.765	2.0071	0.990	2.2725
0.095	0.8148	0.320	1.3789	0.545	1.7239	0.770	2.0132	0.995	2.2782
0.100	0.8353	0.325	1.3877	0.550	1.7308	0.775	2.0193	1.000	2.2839
0.105	0.8550	0.330	1.3964	0.555	1.7376	0.780	2.0254	1.100	2.3971
0.110	0.8739	0.335	1.4050	0.560	1.7444	0.785	2.0315	1.200	2.5683
0.115	0.8922	0.340	1.4136	0.565	1.7512	0.790	2.0375	1.300	2.6179
0.120	0.9098	0.345	1.4221	0.570	1.7589	0.795	2.0435	1.400	2.7264
0.125	0.9269	0.350	1.4306	0.575	1.7647	0.800	2.0495	1.50	2.8337
0.130	0.9434	0.355	1.4390	0.580	1.7714	0.805	2.0555	1.60	2.9401
0.135	0.9595	0.360	1.4473	0.585	1.7781	0.810	2.0615	1.70	3.0458
0.140	0.9751	0.365	1.4556	0.590	1.7848	0.815	2.0675	1.80	3.1508
0.145	0.9903	0.370	1.4638	0.595	1.7914	0.820	2.0735	1.90	3.2553
0.150	1.0051	0.375	1.4720	0.600	1.7980	0.825	2.0795	2.00	3.3594
0.155	1.0195	0.380	1.4801	0.605	1.8046	0.830	2.0855	2.10	3.4631
0.160	1.0335	0.385	1.4882	0.610	1.8112	0.835	2.0915	2.20	3.5564
0.165	1.0473	0.390	1.4962	0.615	1.8178	0.840	2.0975	2.30	3.6694
0.170	1.0608	0.395	1.5041	0.620	1.8243	0.845	2.1035	2.40	3.7720
0.175	1.0740	0.400	1.5119	0.625	1.8308	0.850	2.1095	2.50	3.8745
0.180	1.0869	0.405	1.5197	0.630	1.8373	0.855	2.1154	2.60	3.9768
0.185	1.0995	0.410	1.5275	0.635	1.8438	0.860	2.1213	2.70	4.0789
0.190	1.1119	0.415	1.5353	0.640	1.8503	0.865	2.1272	2.80	4.1808
0.195	1.1241	0.420	1.5430	0.645	1.8567	0.870	2.1331	2.90	4.2826
0.200	1.1361	0.425	1.5507	0.650	1.8631	0.875	2.1390	3.00	4.3843
0.205	1.1479	0.430	1.5583	0.655	1.8695	0.880	2.1449	3.50	4.4891
0.210	1.1595	0.435	1.5659	0.660	1.8759	0.885	2.1508	4.00	5.3958
0.215	1.1709	0.440	1.5734	0.665	1.8823	0.890	2.1567	4.50	5.8993
0.220	1.1821	0.445	1.5809	0.670	1.8887	0.895	2.1625	5.00	6.4120
0.225	1.1931	0.450	1.5884	0.675	1.8951	0.900	2.1683		
0.230	1.2040	0.455	1.5958	0.680	1.9014	0.905	2.1742		

The above table is applicable for any units of measurement, as feet or meters.

The application of Table I and formula (41) will now be illustrated by citing two examples given by Professor Ruehlmann in his Hydromechanik.

Example I. Given a river 80 feet wide and 4 feet deep with a slope $s = 0.000623$. A weir built in this river raises the water 3 feet at the weir site. At what distance back from the weir will the backwater height be 0.25 feet?

Here $\left(\frac{Z}{D}\right) = \frac{3}{4} = 0.75$. From the table, find the value for $f\left(\frac{Z}{D}\right) = 1.9888$. Also $\left(\frac{z}{D}\right) = \frac{1}{4 \times 4} = 0.0625$ for which no value can be found in column 1, hence interpolation by Eq. (45) becomes necessary. Thus:

for $\frac{z}{D} = x_1 = 0.060$, column 2 gives $f(x_1) = y_1 = 0.6376$,

for $\frac{z}{D} = x_2 = 0.065$, column 2 gives $f(x_2) = y_2 = 0.6677$.

By substituting these values in Eq. (45) the value of $f\left(\frac{z}{D}\right)$ corresponding to $\frac{z}{D} = x = 0.0625$ is found as

$$y = f\left(\frac{z}{D}\right) = 0.6526.$$

Having the value of $f\left(\frac{z}{D}\right)$, the required backwater distance is found from Eq. (41) as

$$l = \frac{4}{0.000623}[1.9888 - 0.6526] = 8579 \text{ feet.}$$

Also the total backwater distance as given by Eq. (44) is

$$L = \frac{4}{0.000623} \times 1.9888 = 12{,}770 \text{ feet.}$$

Example II. Given two points A and B, distant 2020 meters apart. What backwater height Z must be produced by a weir at A

such that the backwater height at B is 0.891 m., when the original depth of water is 1.59 m., the fall in the river bed between A

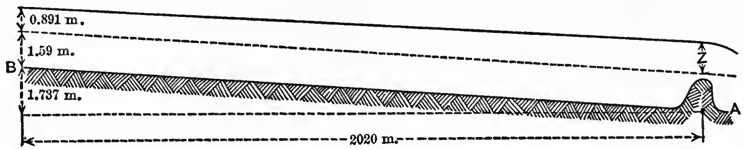


Fig. 23

and B is 1.737 m., and the discharge is 158.52 m.^{3} per second. See Fig. 23.

Then $sl = \frac{1.737}{2020} \times 2020 = 1.737$ and the depth $D = 1.59$ m.; hence $\frac{sl}{D} = 1.0924$ and $\frac{z}{D} = \frac{0.891}{1.59} = 0.560$. Then from Table I, $f\left(\frac{z}{D}\right) = 1.7444$ and from Eq. (43), $f\left(\frac{Z}{D}\right) = 1.0924 + 1.7444 = 2.8368$ for which the table gives very nearly $\frac{Z}{D} = 1.50$, hence

$$z = 1.50 \times 1.59 = 2.385 \text{ m.}$$

CHAPTER VI.

FLOW IN RIVERS AND CANALS.

Derivation of Formulæ for Discharge from Rivers or Lakes into Waterpower Canals and Flumes.

1. *General Discussion.*

THE following formulæ are intended to serve the engineer in designing flumes or canals for manufacturing or water-power purposes. It is proposed to discuss fully the various shortcomings of older formulæ, and to show the general applicability of the foregoing principles and ideas to problems relating to cross-sections, slopes and inlets necessary for the delivery of certain quantities of water through flumes and canals which are supplied either from rivers or lakes.

Ruehlmann, Hydromechanik, p. 439–442, says, for the case of such canal inlets without regulating works, " from a scientific standpoint, the presentation of this problem still lacks all mathematical demonstration, and for practical purposes we must content ourselves with a few observations made by Dubuat." Based on these observations Ruehlmann then offers the following doctrine:

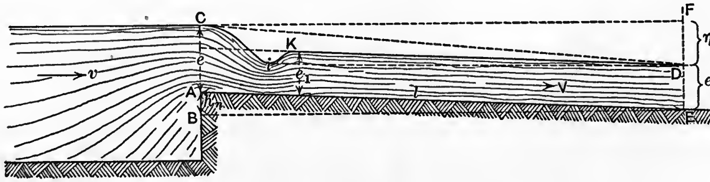


Fig. 24

" The mean velocity and area of flow in any canal or flume of constant width and uniform slope, are so related that velocity

height is equal to the difference in level between the supply reservoir at the inlet and the water in the flume at a point where the velocity of discharge has just become uniform."

Referring to Fig. 24 (which is Ruehlmann's Fig. 167) and calling V the mean velocity of discharge through the section AC, v the velocity of approach, m a coefficient of contraction and other dimensions as indicated on the figure, then, according to Ruehlmann,

$$e - e_1 = \frac{1 \, . \, V^2}{2\, gm^2} - \frac{v^2}{2\, g} \quad \ldots\ldots \quad (46)$$

When v is very small, then

$$e - e_1 = \frac{V^2}{2\, gm^2} \quad \ldots\ldots \quad (47)$$

Ruehlmann then introduces further quantities as follows: l = length of canal at which uniform flow is reached; h_n = absolute slope of the surface KD and of the canal bottom AE; η = actual fall at section DE, below the water level of the inlet; a = sectional area, and w the wetted perimeter at the section DE. Then the required relative slope ratio for the flume, expressed as a function of the head $(e - e_1)$, becomes, fall

$$\frac{h_n}{l} = \frac{\eta - (e - e_1)}{l} \quad \ldots\ldots \quad (48)$$

and

$$\eta = \frac{V^2}{2\, gm^2} + \frac{w}{a}\, l\, \frac{V^2}{k^2} \quad \ldots\ldots \quad (49)$$

wherein k is the experimental coefficient in Chezy's well-known formula for flow in rivers and canals. This formula is

$$v = k\sqrt{\frac{a}{w} \cdot \frac{h}{l}} \quad \ldots\ldots \quad (50)$$

Ruehlmann thus advocates Eqs. (47), (48) and (49), for the solution of all practical problems of the kind here considered.

Regarding these formulæ the criticism is made that they do not take account of section, slope and character of the feeding river, which factors determine the velocity of approach v, as seen from Eq. (46) when solved for V. Thus

$$V = m\sqrt{2\,g\left[(e - e_1) + \frac{v^2}{2\,g}\right]} \quad . \quad . \quad . \quad . \quad (51)$$

by which V is expressed entirely as a function of $(e - e_1)$ and the velocity height $\frac{v^2}{2\,g}$. Formula (46) is, therefore, in error.

The water, in flowing from the section $\overline{AC}$ to the section $\overline{DE}$, is subjected to a variable velocity. Then for the case of an acceleration, the surface might be represented by the dotted line $\overline{CD}$. For a retardation this surface would be the dotted line $\overline{CiD}$, while the special surface $\overline{CiKD} \parallel \overline{AE}$ could scarcely be expected. why not ?

The distance l, required for the velocity of discharge V to become constant and equal to the velocity in the flume, also the total fall η, cannot, in the light of our present knowledge of hydraulics, be computed with any degree of precision. These factors can be ascertained only for an existing flume by means of carefully conducted current meter observations and slope determinations.

The second term $\frac{w}{a}\,l\,\frac{V^2}{k^2}$, of Eq. (49), presupposes that the surface $\overline{KD} \parallel \overline{AE}$ and that the mean velocity over the distance l is constantly equal to V. Both of these presumptions are certainly incorrect and, therefore, Eq. (49) is not strictly reliable.

In presenting the case of a flume with sluice gate at the inlet from the river, Ruehlmann employs the experiments made by Lesbros on small troughs of 0.2 meter in width, placed in the prolonged axis of the supply channel, which latter was 3.68 meters wide, with a supply orifice 0.2 meter wide. The troughs were 3 meters long and the experiments cover a range of slope from 1 : 20 to 1 : 2.9.

Owing to the large dimensions of the supply channel as compared with those of the trough, the velocity of approach was

necessarily very small. Lesbros employs the formula for discharge through a lateral orifice into air and finds for his experiments

$$Q = \mu b\,(H - h)\sqrt{2\,g\left(\frac{H + h}{2}\right)} \quad . \quad . \quad . \quad . \quad (52)$$

wherein H = the fall between the upper level and the sill of the orifice and h = the same fall to the upper edge of the orifice or lower edge of the gate. From this formula Lesbros computes the values for μ which satisfy all the conditions of his experiments.

While it must be admitted that this work might serve a useful purpose within the scope of the observations, yet it is apparent that the above formula (52) is not applicable to the case nor can the discharge coefficients thus found be utilized for computations relating to large flumes or canals.

Ruehlmann (p. 269, § 104) presents another formula for the case illustrated in Fig. 25, where an obstruction is interposed at E, thus slightly damming the water. The area a_1, with water level η above the center of the opening at MN, is supposed to be located

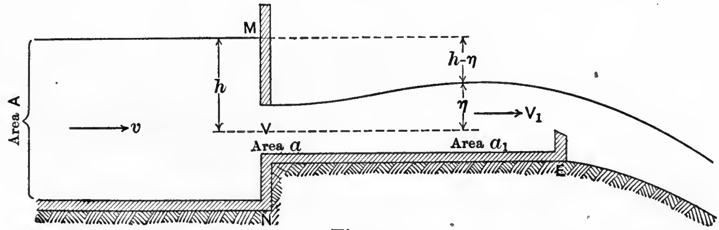


Fig. 25

at such a point where the parallelism of the filaments of flow is again established. Taking areas, velocities and heights as indicated in the figure, then from the principle of the *conservation* of *energy*, the following equation may be written out :

$$\left.\begin{aligned} gM\,(h - \eta) &= \frac{M}{2}\,(V_1^{\,2} - v^2) + \frac{M}{2}\,(V - V_1)^2 \\ \text{or} \qquad (h - \eta) &= \left(\frac{V_1^{\,2}}{2\,g} - \frac{v^2}{2\,g}\right) + \frac{(V - V_1)^2}{2\,g} \end{aligned}\right\} \quad . \quad . \quad (53)$$

Herein M represents the mass of water passing any section per second.

For the case where A is very large in comparison with a, Poncelet first developed the following formula from Eq. (53):

$$Q = a_1 \sqrt{\frac{2\,g\,(h - \eta)}{1 + \left(\frac{a_1}{\alpha a} - 1\right)^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

In the preceding chapters, on the derivation of new weir formulæ, it is conclusively shown that the energy stored in a moving mass of water is not expended entirely in the production of flow through a contracted orifice, but that a large portion of that energy is lost as impact against all objects causing obstruction or contraction, and hence the principle of the *conservation of energy* is not applicable to a mass of water approaching a sluice or orifice.

By inspection of Eq. (53) it is readily seen that the difference in height $(h - \eta)$ furnishes the pressure height producing the acceleration necessary to increase v to the value of V_1 and also to overcome the resistance encountered in reducing V to the value V_1. Also, the expression $\left(\frac{V_1^2}{2\,g} - \frac{v^2}{2\,g}\right)$ is incorrect because the velocities V_1 and v are in no wise related, since v is first accelerated to the value V and is then retarded to the value V_1. Furthermore, the pressure height necessary for the acceleration of v becomes greater in proportion to the increased resistance to flow through the area a due to details of design. And lastly, the velocity V_1 is not dependent entirely on the pressure height $(h - \eta)$, but largely on the slope, width, depth and frictional resistances in the discharging flume. Hence for two different cases even though v and V_1 remained the same, the controlling height $(h - \eta)$ might be very different, a condition which cannot be rectified in the equation as it stands.

The last expression $\left(\frac{V - V_1}{2\,g}\right)^2$ from Eq. (53), represents the pressure height corresponding to a retardation $V - V_1$ for the

case of a sudden contraction in a closed pipe, according to Carnot's principle. However, this principle is not applicable to open flumes, because the water is not confined on all sides and at least a portion of the head $(h - \eta)$ can be expended in raising the level of the discharge surface. The velocity V is not suddenly changed to V_1 but the change is gradual and hence little of the *vis viva* is lost. All this is contrary to the underlying principles of the formula, and hence both Eqs. (53) and (54), are wrong and irrational.

2. *Proposed General Solution.*

It is thus seen that no reliable formulæ exist for any of the cases of flow just discussed.

However, such problems can be solved with reasonable accuracy by employing the formulæ previously derived for submerged and sluice weirs in combination with such formulæ as those of Ganguillet and Kutter for flow of water in rivers and canals.

Thus the problem presented in the above Fig. 25, may be solved with the aid of Eqs. (36), by taking the highest point of the discharge surface in the flume as the governing level of the lower pool and assuming the velocity of flow for this point equal to V_1.

In the derivation of the previous formulæ it was seen that the water in the lower pool influences the discharge quantity by exerting a hydrostatic counterpressure, and also by producing a suction on the area of flow. Hence the new formulæ, which make allowances for these various conditions, are in every way applicable to a flume supplied from a lake or river, provided the depth and velocity of the water discharged through the flume is previously known.

By referring to Eqs. (36), and the reasoning there followed, it is understood that the total head H_2, between the upper and lower pools adjacent to a sluice gate, is not the governing factor determining the discharge through the flume, but that the quantity of water discharged is determined by the dimensions and slope of the flume or canal. Hence, the formulæ for flow in rivers and canals must be employed for the purpose of deciding the necessary dimen-

sions and slope to deliver the required quantity, and this result is then used in the new formulæ for flow through regulating works.

From many gagings which have been conducted in the past, formulæ of more or less accuracy are now available for flow through rivers, creeks, canals and flumes. These express the velocity or quantity of flow in terms of cross-section, slope and friction.

For steep slopes in canals or flumes, Chezy's formula is probably most applicable. This formula, as revised by Bazin, in 1897, is

$$\left.\begin{aligned} Q = AV = AC\sqrt{rs}, \quad \text{where } C &= \frac{87}{0.552 + \dfrac{m}{\sqrt{r}}} \text{ for feet,} \\ \text{and} \qquad C &= \frac{87}{1 + \dfrac{m}{\sqrt{r}}} \text{ for meters} \end{aligned}\right\} \tag{55}$$

and V = velocity in feet per second; r = mean hydraulic radius in feet $= \dfrac{A}{w}$ = area of flow divided by the wetted perimeter; s = slope = fall, in feet, divided by length, in feet, over which the fall occurs.

The experience coefficient m, depending on the roughness of the wetted surface, is given by Bazin (1897) as follows:

For smooth cement or planed wood	$m = 0.06$
For rough planks and brick	$m = 0.16$
For masonry	$m = 0.46$
For regular earth beds and slopes	$m = 0.85$
For canals in good order	$m = 1.30$
For canals in very bad order	$m = 1.75$

These values are applicable to dimensions in feet and meters alike.

For tabulated values of C, corresponding to values of r from one to ten, see tables 46 and 47, p. 565, Merriman's Hydraulics, 1904.

Kutter's formula for the value of C in Eqs. (55) is probably

the best in the present state of our knowledge for small slopes. This value, for r and V in feet, is

$$C = \frac{\frac{1.811}{n} + \left(41.65 + \frac{0.00281}{s}\right)}{1 + \frac{n}{\sqrt{r}}\left(41.65 + \frac{0.00281}{s}\right)} \quad . \quad . \quad . \quad . \quad (56)$$

For r and V in meters the formula becomes

$$C = \frac{\frac{1}{n} + \left(23 + \frac{0.00155}{s}\right)}{1 + \frac{n}{\sqrt{r}}\left(23 + \frac{0.00155}{s}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

The values of n in both formulæ (56) and (57) are alike, and according to Kutter they are as follows:

For well planed timber	$n = 0.009$
For neat cement	$n = 0.010$
For cement with one third sand	$n = 0.011$
For unplaned timber	$n = 0.012$
For ashlar and brick work	$n = 0.013$
For unclean sewers and conduits	$n = 0.015$
For rubble masonry	$n = 0.017$
For canals in very fine gravel	$n = 0.020$
For canals and rivers free from stones and weeds	$n = 0.025$
For canals and rivers with some stones and weeds	$n = 0.030$
For canals and rivers in bad order	$n = 0.035$

Tabulated values for C, by Kutter's formulæ, are given in Tables 44 and 45, p. 564, Merriman's Hydraulics, 1904; also in Trantwine, but most completely in Bellasis, p. 183 *et seq.*

With the use of either Bazin's or Kutter's formulæ then, the values of Q and V can be determined for any particular flume or canal. Or, suppose it is desired to design a flume to deliver an assigned quantity of water Q, then the required slope, the area

and the velocity may be so chosen as to fulfil the conditions for discharge.

After thus assigning dimensions and slope to the flume, then the discharge area and head, necessary in the regulating works at the flume inlet, can be determined from one of the previously given weir or sluice gate formulæ. Should the head H_2, between the supply canal and the flume, be given, then the discharge area alone will require dimensioning.

If the problem be stated thus: Given a regulating sluice gate of certain dimensions at the inlet of a flume, the slope s and width B of the latter being also given, to find the depth d, and quantity of discharge Q, through the flume, then the following solution is proposed: First compute the discharge through the sluice gate, using one of the formulæ 30, 36 or 38 appropriate to the case in hand, and assuming that the discharge takes place into the air without any water in the flume. This value is naturally excessive and hence it can be reduced somewhat to obtain the first approximate value Q_1, which, when inserted into the formulæ (55), will give preliminary values A_1 and V_1 and thus obtain a depth $d_1 = \frac{A_1}{B}$. Now by inserting the values A_1 and V_1 into the original sluice gate or weir formula first employed, a second quantity Q_2 is found as a second approximation.

By a repetition of this process, using Q_2 in the Chezy formula to obtain A_2 and V_2 and finding $d_2 = \frac{A_2}{B}$, and then employing A_2 and V_2 in the sluice gate formula to find a value Q_3, finally obtain such a value Q as will satisfy both equations, from which

$$d = \frac{Q}{VB} = \frac{A}{B}$$

is found with reasonable accuracy.

Perhaps a better way to carry on this successive approximation is to plot the curves representing the general relation between Q and d, for each of the two formulæ, and these two curves will

intersect in a common point, the Q and d of which will satisfy both equations.

The foregoing discussion has been confined to cases where the axis of the flume is in continuation of the supply channel axis and the water in the flume extends upstream to the regulating works.

3. *New formulæ for the case when all the available water of a lake or river should be diverted into a lateral canal or flume by a dam built normally to the river.*

A factory flume is required to furnish an assigned quantity Q, to be supplied from a river by interposing a dam. The flume is supposed to have its inlet just above the weir. This will require assigning dimensions to the regulating gate at the flume inlet and also to the flume itself and then give the flume such a slope as is necessary to deliver the quantity Q.

Formulæ for the solution of such problems have not hitherto been proposed, hence the following discussion is offered.

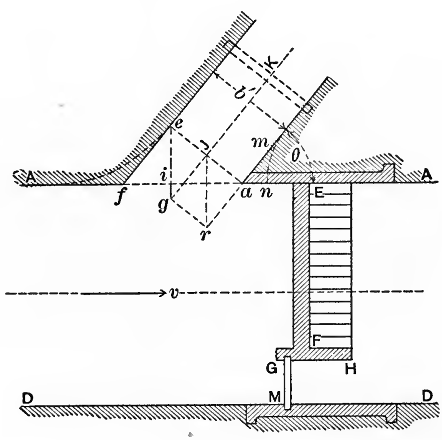


Fig. 26

In Fig. 26, let $\overline{EF}$ represent a spillway, with regulating gate at $\overline{GM}$, both normal to the flow of the river. $\overline{JK}$ is a lateral diversion channel making the angle θ with the axis of the river.

A longitudinal section along $\overline{AA}$, showing the flume inlet $\overline{f_1 f a_1 a}$, is given in Fig. 27. The first case to receive consideration is when the water level of the river is even with the crest of the dam at E, and the entire discharge Q passes out through the flume.

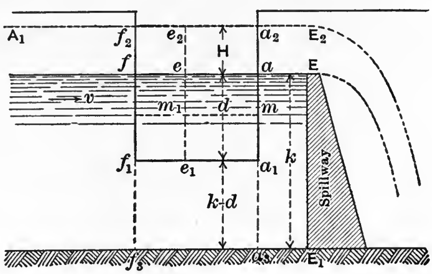


Fig. 27

The hydrostatic pressure against the flume area is taken the same as against the dam, and the hydrodynamic pressure as found from Eq. (13) $= p_5 = \gamma \frac{v^2}{g} Bk$ which (per unit of area) becomes $p_6 = \gamma \frac{v^2}{g}$.

Using the dimensions indicated in the two figures, then $\overline{ai} = \overline{ae} \sin \theta$, and the dynamic pressure on the area ai is all that is transmitted to the discharge area. Hence the pressure on this entire area

$$= p_7 = \gamma \frac{v^2}{g} b^1 d \sin \theta.$$

Let the line $\overline{Jr}$ represent the magnitude of p_7, which may be resolved into components $\overline{Jg}$ and $\overline{Ja}$. Then the component $\overline{Jg}$, normal to $\overline{ae}$, is found from:

$$\overline{Jg} = \overline{Jr} \cos (90 - \theta) = \gamma \frac{v^2}{g} b' d \sin^2 \theta,$$

which may be represented by a rectangular prism of height d, in front of the discharge area. This prism will have a breadth b', and length $\frac{v^2}{g} \sin^2 \theta$.

The hydrodynamic pressure p_6 of the approaching water, acting normally to the banks, and hence deflected into the direction of the flume, may be regarded active over the area $b'd$ and in the direction $\overline{JK}$.

The hydrodynamic pressure deflected by the weir against the flume inlet, by analogy to Eq. (11), becomes

$$p_4 = \gamma d \frac{v^2}{g} \frac{0.5\, b'}{2} = 0.25\ \gamma d b' \frac{v^2}{g}$$

because H is now d, $\frac{B-b}{2}$ becomes $0.5\, b'$, and $\phi = 90°$. This pressure p_4, being equally distributed over the area $b'd$, may again be represented by a rectangular prism in front of the discharge area. Assume its height d, breadth b' and length $0.25 \frac{v^2}{g}$.

The water flowing in the lower section of the river, exerts a dynamic pressure which is transmitted normally and horizontally against the flow area, and may be represented by a triangular prism of height $(k - d)$, a width $= b' \sin\theta$, and a bottom length β,

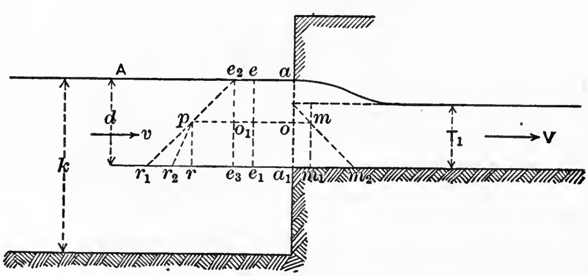


Fig. 28

which may be determined from Eq. (16) as follows: see Figs. 27 and 28.

From Eq. (16) $\beta = \frac{2\, v^2 B k}{b' g H} \cos^2 \frac{\psi}{2}$, in which the area $b'H$ is represented by $\overline{oa_1} = T_1 - \frac{nV^2}{2\,g}$, in Fig. 28, and $\frac{nV^2}{2\,g}$ is equal to

$\overline{om}$. Also the area Bk now becomes $(k - d)\sin\theta$, and since the resultant in the direction $\overline{JK}$ is wanted, the whole expression, for a unit width, finally becomes

$$\beta = \frac{2\,v^2(k-d)}{g\left(T_1 - \dfrac{nV^2}{2\,g}\right)}\sin^2\theta\cos^2\frac{\psi}{2}\quad . \quad . \quad . \quad . \qquad (58)$$

All of these prisms are now combined in Fig. 28, as follows: make $\overline{ae} = \dfrac{v^2}{g}\sin^2\theta$; $\overline{ee_2} = 0.25\dfrac{v^2}{g}$; $\overline{om} = \dfrac{nV^2}{2\,g}$, and $\beta = \overline{rr_2}$; then the area e_2e_3pr represents the hydrostatic pressure and the final hydraulic pressure, active on the area $\overline{a_1ae_1e}$, is represented by the area $\overline{ae_2pr_2a_1}$.

The resultant unit pressures along any horizontal filament of the pressure area are thus easily found from Fig. 28, and the following values are now derived:

The hydraulic pressure along the surface filament is

$$S = \overline{ae} + \overline{ee_2} = \frac{v^2}{g}(0.25 + \sin^2\theta)\quad . \quad . \quad . \qquad (59a)$$

The hydraulic pressure along $\overline{op}$, where the hydrostatic counter-pressure of depth T_1 becomes active, is

$$S_1 = \overline{oo_1} + \overline{o_1p} = S + d - \overline{oa_1} = S + d - \left(T_1 - \frac{nV^2}{2\,g}\right) \qquad (59b)$$

The hydraulic pressure at the bottom of the flume is

$$S_2 = \overline{op} + \overline{rr_2} = S_1 + \beta = S_1 + \frac{2\,v\,(k-d)}{g\left(T_1 - \dfrac{nV^2}{2\,g}\right)}\sin^2\theta\cos^2\frac{\psi}{2} \qquad (59c)$$

Employing these values of S, S_1, and S_2, the discharge into free air, through the upper portion $\overline{ao}$ of the section, may now be found from Eq. (19), in which $H = d - T_1 + \dfrac{nV^2}{2\,g}$,

and
$$S_1 - S = d - T_1 + \frac{nV^2}{2\,g},$$

hence
$$Q_1 = \tfrac{2}{3}\,\mu b'\sqrt{2\,g}\,[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}]\quad . \quad . \quad . \quad . \qquad (59d)$$

In like manner the discharge through the submerged portion $\overline{oa_1} = T_1 - \frac{nV^2}{2g}$, is found from Eq. (19) by making $H = \overline{oa_1}$, then

$$Q_2 = \tfrac{2}{3}\mu_1 b'\sqrt{2g}\left(\frac{T_1 - \frac{nV^2}{2g}}{S_2 - S_1}\right)[S_2^{\frac{3}{2}} - S_1^{\frac{3}{2}}] \quad . \quad . \quad (59e)$$

The total flow through the section $b'd = \overline{a_1 a e_1 e}$ is then

$$Q = Q_1 + Q_2$$

$$= \tfrac{2}{3}\, b'\sqrt{2g}\left[\mu\,(S_1^{\frac{3}{2}} - S^{\frac{3}{2}}) + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\left(\frac{S_2^{\frac{3}{2}} - S_1^{\frac{3}{2}}}{S_2 - S_1}\right)\right] \qquad (59f)$$

Since the Chezy formula also furnishes a value for the uniform flow through the flume, then for any given case, for which the dimensions are assigned or determined from local conditions, the three unknowns can be found as previously indicated.

When the flume is supplied from a lake, or river of sluggish flow, then the velocity of approach can be disregarded and the above formulæ (59) reduce to simpler forms. Thus by making $v = 0$, then

$S = 0$, $S_2 = S_1 = d - T_1 + \frac{nV^2}{2g}$, and approximating Q_2 as was done in Eq. (28*f*), then

$$Q = b'\sqrt{2g\left(d - T_1 + \frac{nV^2}{2g}\right)}\left[\frac{2}{3}\mu\left(d - T_1 + \frac{nV^2}{2g}\right) + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\right] \quad . \quad . \quad . \quad . \quad (60)$$

Formulæ (59) will also apply to the case illustrated in Fig. 9, because when no discharge takes place over the spillway, the approaching flow exerts equal pressure in all directions.

The application of Eqs. (59) will now be illustrated by solving a problem such as is likely to occur in practice. The dimensions are all taken in the metric system, merely as a matter of convenience.

Problem. Given a river 10 m. wide, discharging 6 m.^{3} per second. It is desired to build a factory flume having its inlet just above a spillway which is to be built normally to the river and of 3 m. height with vertical upstream face. The flume is to make an angle of 45 degrees with the axis of the river and must deliver the full river discharge to a factory, 2000 m. distant, when the water stands level with the top of the spillway.

The section of the flume may be chosen as trapezoidal with 1:1 slopes and of such area that when the water is flowing with velocity $V = 1$ m. per second, it shall deliver the required Q when the depth of flow T_1, in the flume, is 1 m.

The question then resolves itself into finding the width b', and depth d of the flume inlet, also the bottom width b'', and slope s of the flume, necessary to discharge 6 m.^{3} per second.

The terms in Eqs. (59) will then have the following values: $B = 10$ m., $Q = 6$ m.^{3}, $k = 3$ m., $v = \dfrac{Q}{Bk} = 0.2$ m. per second, $V = 1$ m. per second, $T_1 = 1$ m., and $\theta = 45°$. Also $\psi = 90°$ when the river bank at the flume inlet is made vertical by a retaining wall.

The four unknown quantities are b', d, b'' and s, but as only three equations are available it will be necessary to make $b' = b''$ which is equivalent to making the bottom width of the flume equal to the uniform width of the flume inlet.

From $Q = T_1(b' + T_1)\,V$ the value $b' = \dfrac{Q}{T_1 V} - T_1 = 5$ m. $= b.''$

The slope s, of the flume, is found by Chezy's formula, Eq. (55), wherein $r = \dfrac{\frac{5+7}{2} \times 1}{7.828} = 0.7665$ m. and $\sqrt{r} = 0.875$. Taking $m = 1.3$, $C = 35$, then $\sqrt{s} = \dfrac{Q}{AC\sqrt{r}} = \dfrac{6}{6 \times 35 \times 0.875}$ $= 0.0326$ or $s = 0.001063$, and the total fall required in the flume over the distance of 2000 m. will be 2.12 m. By building

the flume rectangular in section, with cement walls and bottom, to reduce friction, and maintaining the area 6 m.^{3} = 1×6, and velocity $V = 1$ m. per second, this total fall could be reduced to about 0.4 m.

Lastly, to find d from the formulæ (59) the coefficients μ and μ_1 must be known. In want of better data, these values are assumed each equal to 0.6, a value which is based on the experiments of Francis.

Now substituting all of the foregoing values into the Eqs. (59) it will be found that the value $d = 1.136$ m. satisfies the three equations, making $Q_1 = 0.64$ m.^{3}, $Q_2 = 5.36$ m.^{3}, and $Q = Q_1 + Q_2 = 6$ m.^{3}

Hence, if the flume inlet be made 5 m. wide $\times$ 1.136 m. deep, thus making the water in the flume $d - T_1 = 0.136$ m. lower than in the river, then the flume will discharge the required quantity $Q = 6$ m^3.

This example illustrates the points previously brought out in the present chapter, viz., that the slope of the flume is not a function of $d - T_1$, but depends on the section and roughness of the flume.

4. *New formulæ for the case when a portion of the flow in a river should be diverted and the remainder be discharged over a weir built normally to the river.*

This case is exactly like the preceding, only that a portion of the water is discharged over the weir along a profile $\overline{A_1E_2}$, Fig. 27, such that the depth H over the crest of the weir, may be maintained aside from furnishing a stipulated flow through the power canal. Q will here represent the quantity to be diverted for power purposes.

The problem of finding the hydraulic pressure active on the flume area $\overline{a_1a_2f_1f_2}$ is very difficult, owing to the interference of cross currents, the retarding effect of which would depend largely on the relative quantities of flow in the two directions. Hence nothing better than an approximation could be expected.

The hydrostatic and hydrodynamic pressures exerted against the area of flow into the flume will be found as in the previous solution but modified as follows:

1. The flow of approach will not expend its entire hydrodynamic pressure $\frac{v^2}{2g}$ on the area of flow because the direction of the river current is past this area.

2. For this same reason, no hydrodynamic pressure will be deflected from the river banks against the area of flow.

3. That portion of the hydrodynamic pressure resulting from the flow against the area $\overline{a_1a_3f_1f_3}$, Fig. 27, will now be divided so that in the present case this pressure will be assumed only half as great as it was with the water dammed.

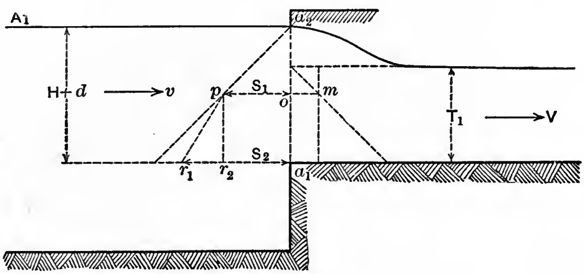


Fig. 29

These several conditions are shown on the profile, Fig. 29, where the graphic areas represent the pressures, exerted by various causes, against the area of flow. Figs. 26 and 27 still apply.

The depth of flow into the flume inlet is now $d + H$, Fig. 27; the suction on the flow area produced by the water in the flume is represented by $\overline{om} = \frac{nV^2}{2g}$, and the uniform depth of flow in the flume is T_1. The pressure height $\overline{a_2o}$ is

$$\overline{a_2o} = d + H - \left(T_1 - \frac{nV^2}{2g}\right)$$

from which the quantity Q_1, flowing through the discharge area $\overline{a_2 o}$ into free air, is found as

$$Q_1 = \tfrac{2}{3}\mu b' \cdot \overline{a_2 o}\sqrt{2g \cdot \overline{a_2 o}} = \tfrac{2}{3}\mu b' \sqrt{2g}\left(d + H + \frac{nV^2}{2g} - T_1\right)^{\frac{3}{2}} \quad (61)$$

In the derivation of Eq. (15), the hydraulic pressure exerted by the water flowing below the discharge section against this section, was found to be $p_6 = \gamma \frac{v^2}{g} Bk \cos^2 \frac{\psi}{2}$. For the present case $k - d$ must be substituted for k, and $b \sin \theta$ for B. Also the hydrodynamic pressure becomes $\frac{v^2}{2g}$ and retaining ψ as the angle of slope of the river bank below the flume inlet, then the value p_6 becomes

$$p_6' = \gamma \frac{v^2}{2g}(k - d)\, b' \sin\theta \cos^2 \frac{\psi}{2}.$$

Since this pressure may be assumed as varying with the depth, being a maximum at the bottom of the flume $\overline{a_1 e_1}$, and zero at $\overline{pm}$, it may be represented by a triangular prism of base β, height $\frac{\gamma}{2}\left(T_1 - \frac{nV^2}{2g}\right)$ and length b'. The previous equation for p_6' gives

$$\beta = \frac{v^2}{g}\left(\frac{k - d}{T_1 - \frac{nV^2}{2g}}\right) \sin\theta \cos^2 \frac{\psi}{2} = \overline{r_2 r_1} \text{ in Fig. 29.}$$

Hence the trapezoid $\overline{opa_1 r_1}$ represents the total effective hydraulic pressure against the area of discharge $\overline{oa_1}$ into the flume. The pressure S_1 along the filament $\overline{op}$ and the pressure S_2 along the bottom of the flume $\overline{a_1 r_1}$ are evaluated as

$$S_1 = d + H + \frac{nV^2}{2g} - T_1 \quad \dots\dots\dots \quad (62a)$$

$$S_2 = S_1 + \beta = S_1 + \frac{v^2}{g}\left(\frac{k - d}{T_1 - \frac{nV^2}{2g}}\right) \sin\theta \cos^2 \frac{\psi}{2} \quad \dots \quad (62b)$$

The quantity of discharge Q_2 through the area $\overline{oa_1}$, Fig. 29, is found from Eq. (19) by making the head equal to $\left(T_1 - \frac{nV^2}{2g}\right)$, thus:

$$Q_2 = \frac{2}{3}\mu_1 b' \sqrt{2g}\left(\frac{T_1 - \frac{nV^2}{2g}}{S_2 - S_1}\right)[S_2^{\frac{3}{2}} - S_1^{\frac{3}{2}}] \quad . \quad . \quad (62c)$$

Hence $\quad Q = Q_1 + Q_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62d)$

Should v and $k - d'$ be small, then the value for Q_2 might be reduced to the form

$$Q_2 = \mu_1 b' \left(T_1 - \frac{nV^2}{2g}\right)\sqrt{2gS_1} \quad . \quad . \quad . \quad . \quad . \quad (62e)$$

These formulæ (62), in combination with the Chezy formula, furnish a solution for problems of the kind previously treated when only three unknown quantities remain to be determined.

5. *New formulæ for the case when a portion of the flow in a river should be diverted, and the remainder be discharged over a weir built diagonally across the river.*

This case is illustrated in plan, Fig. 9, while Fig. 27 will again serve for longitudinal section. Finally Fig. 30 shows the pressure areas active against the discharge area of the flume.

The water in the river approaches with a surface $\overline{A_1E_2}$ and the depth on the crest of the weir is again taken equal to H, see Fig. 27.

From Fig. 9, the total hydrodynamic pressure of the flow, over the height k of the weir, is $\gamma Bk\frac{v^2}{g}$, represented in the figure by the magnitude $\overline{e_1a_1}$, which is resolved into components $\overline{e_1g_1}$ and $\overline{e_1f_1}$, respectively normal and parallel to the face of the weir. Then $\overline{e_1g_1}$ is expended against the weir and $\overline{e_1f_1}$ becomes effective on the discharge through the flume in an amount

$$\overline{a_1e_1}\cos\phi = e_1f_1 = \gamma\frac{v^2}{g}Bk\cos\phi,$$

directed towards the bank $\overline{LM}$.

That portion p of the pressure $e_1 f_1$, Fig. 9, which is active over the height d, Fig. 27, of the flume inlet, is found from Fig. 26 as

$$p = \gamma \frac{v^2}{g} db' \frac{\sin \phi}{\sin \theta},$$

by observing that $\overline{LM} = \frac{B}{\tan \phi}$ and $\overline{MN} = \frac{b'}{\sin \theta}$ from Fig. 9. But the pressure exerted normally to b', Fig. 26, is

$$p \cos (\theta - \phi) = \gamma \frac{v^2}{g} db' \frac{\sin \phi}{\sin \theta} \cos (\theta - \phi).$$

Now since p is uniformly distributed in the vertical direction, it may be represented by a rectangular pressure area of

length b', height d, and breadth $\beta = \frac{v^2}{g} \cdot \frac{\sin \phi}{\sin \theta} \cos (\theta - \phi)$ (63)

acting against the flume inlet.

The hydrodynamic pressure exerted against the weir surface below the flume inlet is expressed by

$$p_1 = \gamma \frac{v^2}{g} (k - d) B \cos \phi,$$

and the portion of p_1 expended over the width of the inlet $\overline{a_3 f_3}$, Fig. 27, against the river bank is

$$p_2 = \gamma \frac{v^2}{g} (k - d) b' \frac{\sin \phi}{\sin \theta}.$$

This pressure p_2 acting against an inclined bank, making the angle ψ with the horizontal, causes a pressure against the flume discharge area tending to accelerate the discharge velocity. The amount of this effect is given from Eq. (15) as

$$p_6 = \gamma \frac{v^2}{g} (k - d) b' \frac{\sin \phi}{\sin \theta} \cos^2 \frac{\psi}{2}.$$

Since p_6 is not uniformly active but creates maximum effects at the bottom of the flume, it is again represented by a triangular

pressure area against the discharge opening. This pressure prism will have a length b', height $\left(T_1 - \frac{nV^2}{2g}\right)$ and base β_1 found from

$$\gamma \frac{b'}{2}\left(T_1 - \frac{nV^2}{2g}\right)\beta_1 = \gamma \frac{v^2}{g}(k-d)\, b' \frac{\sin\phi}{\sin\theta} \cos^2\frac{\psi}{2},$$

whence

$$\beta_1 = \frac{2v^2}{g} \cdot \frac{(k-d)}{\left(T_1 - \frac{nV^2}{2g}\right)} \frac{\sin\phi}{\sin\theta} \cos^2\frac{\psi}{2} \quad . \quad . \quad . \quad (64)$$

The pressure areas, effective against the flume inlet, are graphically indicated in Fig. 30.

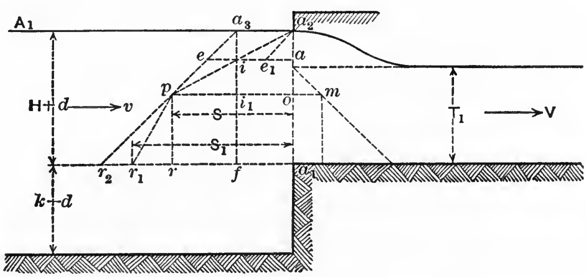


Fig. 30

The suction produced by the discharge velocity V on the section $\overline{a_1a_2}$ is represented by $\overline{om} = \frac{nV^2}{2g}$. The area $\overline{aia_1f}$ represents the hydrodynamic pressure active on the flume discharge area, where $\overline{ai} = \overline{a_1f} = \beta$ from Eq. (63). The trapezoid $\overline{a_3frp}$ represents the total hydrostatic pressure plus the suction $\frac{nV^2}{2g}$ active on the discharge area. Also, making $rr_1 = \beta_1$, the triangle $\overline{prr_1}$ becomes the pressure area resulting from the flow below the bottom of the flume. Finally, transferring the triangle $\overline{a_3ei}$, representing the hydrostatic pressure due to the head H, to the position $\overline{a_2e_1a}$ in front of the discharge area, the whole hydraulic pressure will be represented by the figure $\overline{a_1r_1pee_1a_2}$.

Since the pressure on the part section $\overline{a_2 o}$, Fig. 30, undergoes an abrupt change $\overline{e_1 e}$, which could not exist in reality and would necessitate some further complications in the resulting formulæ, therefore, the pressure area $\overline{opee_1 a_2}$ may be replaced by the triangle $\overline{opa_2}$. In this triangle the height and base have the following values:

$$\overline{a_2 o} = \left[d + H + \frac{nV^2}{2g} - T_1\right] \text{ and } \overline{op} = S = \overline{oi_1} + \overline{i_1 p},$$

making

$$S = \beta + \overline{i_1 p} = \frac{v^2 \sin\phi}{g \sin\theta} \cos(\theta - \phi) + \left[d + H + \frac{nV^2}{2g} - T_1\right] \quad . \quad (65a)$$

The quantity of discharge Q_1 through the part section $\overline{a_2 o}$ will then be

$$Q_1 = \frac{2}{3} \mu b' \left[d + H + \frac{nV^2}{2g} - T_1\right] \sqrt{2gS} \quad . \quad . \quad (65b)$$

Also, the resulting pressure $\overline{a_1 r_1} = S_1 = \overline{a_1 r} + \overline{rr_1} = S + \beta_1$, active at the bottom of the flume, will be

$$S_1 = S + \frac{2v^2}{g}\left[\frac{k - d}{T_1 - \dfrac{nV^2}{2g}}\right]\left(\frac{\sin\phi}{\sin\theta}\right)\cos^2\frac{\psi}{2} \quad . \quad . \quad (65c)$$

The quantity of flow Q_2 through the lower portion $\overline{a_1 o}$ of the flume inlet will then become, after inserting the proper value for H into Eq. (19),

$$Q_2 = \frac{2}{3} \mu_1 b' \sqrt{2g} \left[\frac{T_1 - \dfrac{nV^2}{2g}}{S_1 - S}\right] (S_1^{\frac{3}{2}} - S^{\frac{3}{2}}) \quad . \quad . \quad (65d)$$

The total discharge into the flume is then

$$Q = Q_1 + Q_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65e)$$

Should the angles ϕ and θ be equal, or nearly so, and the angle ψ be made 90 degrees, also making $v = 0$, then these formulæ (65) can be very much simplified as was done for Eqs. (60).

By combining these formulæ with Chezy's formula, three unknown quantities may again be found as illustrated in previous examples.

If a sluice gate were placed at the inlet to the flume then the hydrostatic and hydrodynamic pressures resulting from the back water in the river against the gate, may be found in the manner previously described, finally computing the discharge through the gate from one of the formulæ (36) or (38) according to the case in hand.

This closes the theoretical portion of the present treatise.

CHAPTER VII.

EMPIRIC COEFFICIENTS.

1. Introductory.

In the foregoing theoretical chapters no consideration was given to the various empiric coefficients employed. This subject is one of vital importance to the usefulness of any formulæ, which latter can never be made to include all the disturbing elements always present when dealing with river hydraulics. The best that can ever be hoped for is the adaptation of rational formulæ to the observed facts and to correct the shortcomings by the introduction of numerical coefficients.

The effort here made in the direction of furnishing the rational forms for many of the complex problems frequently encountered in practice, by taking into account nearly all of the variable factors, should merit the appreciation of every hydraulic engineer.

The attempt will now be made to evaluate the coefficients for these new formulæ by employing all available experiments bearing directly on the subject. However, suitable and reliable experiments of the kind in question are not numerous. The very valuable and painstaking hydraulic experiments made by Messrs. A. Fteley and Frederic P. Stearns and published in 1883, Trans. Am. Soc. C. E., Vol. 12, and the classic "Lowell Hydraulic Experiments," 1855, by Mr. James B. Francis, together with some more recent, but limited experiments, constitute about all the available data. Thanks then to the existence of these experiments, some reliable values for μ in the new formulæ have been found, although in many of the cases theoretically discussed, no empiric data are as yet at hand, from which to deduce coefficients.

The rational formulæ, previously derived, will still give values

in excess of the actual when the coefficients are neglected. Hence these coefficients are really reduction factors to reduce theoretical to real discharge. The causes producing this effect may be thus summarized:

a. The loss in its *vis viva* of the flow of approach due to impact and deflections near and at the discharge area.

b. The friction and cohesion around the wetted surface at the discharge section, producing a retardation in the discharge velocity.

c. The contraction in, and immediately adjacent to, the discharge section. This contraction, which is probably the most significant of the three named effects, is variable and depends on the dimensions of the overfall, on the interrelation of these dimensions, on the canal and its configurations near the weir, and on the velocity and depth of the flow of approach. In the following, two forms of contraction are distinguished, complete and partial. *Complete contraction* takes place when the discharge section is suddenly reduced on all sides of the orifice, as for discharge from a lateral orifice. *Partial contraction* occurs when the approaching flow is confined only on three sides, and the discharge section is contracted on three sides.

The total correction for all the foregoing retarding influences will then be made by introducing an empiric reduction factor. This factor for discharge into free air will be called μ and for discharge through an entirely submerged section it will be called μ_1. For any given set of experiments the values of either or both of these coefficients are then determined by inserting all the observed quantities into the appropriate formulæ to find the theoretical quantity of discharge, which latter divided into the observed discharge gives the corresponding μ. This method will be applied in the following to all available observations covering the widest possible range in weir and canal dimensions.

For the most part the observations from the experiments of Messrs. Francis, Fteley, and Stearns, have been used. Aside from these the Cornell University experiments on standard sharp crested weirs made in 1899, and a few old experiments made by

Lesbros, in 1829 to 1834, and some made by Boileau, in 1845, were employed merely to extend the range of the former observations.

It should be noted that the values μ are alike for United States and metric units, so long as all dimensions are expressed in terms of one such unit only. That is, μ is merely a ratio between two volumes, which ratio remains the same independent of the unit chosen.

Regarding the coefficient in the formula for discharge through lateral orifices it should be mentioned that the usually accepted value is really the product of the numerical coefficient $\frac{2}{3}$ and μ, so that the theoretical part of the formula is

$$Q = \tfrac{2}{3}\, bH \sqrt{2\, gH}.$$

However, the old values have been so widely adopted that a change would greatly confuse matters; therefore, in all the following tables μ and $\frac{2}{3}\mu$ have been tabulated together.

2. Complete Overfalls.

(a.) *Coefficients* μ_s *for Eqs.* (19) *to* (22), *for weirs normal to the channel and no wing walls, hence* $B = b$. *See observations, Tables* II *and* III.

In all these observations $B = b$, $\phi = 90$ degrees, and $\psi = 90$ degrees except for observations 46 to 50, where $\psi = 20$ degrees and a wide crest was used. The values for $\frac{2}{3}\mu$ and μ are found from Eqs. (19) to (22) and inserted in these Tables II and III.

Since it is necessary to provide values for μ for any particular problem in hand, the following empiric formula (66) is offered for its computation. In this formula, μ is made to depend on b, k and H, and a constant as expressed by Eq. (66),

$$\frac{2}{3}\mu = \left[\alpha + \beta\left(\frac{H}{H+k}\right) + \frac{\gamma}{H} + \delta b\right]\rho = \frac{2}{3}\mu_s\rho \quad . \qquad (66)$$

The factor ρ is the experience factor for *any crest* compared with the *standard sharp crest*, and the quantity inside the parenthesis will represent $\frac{2}{3}\mu_s$ which is the special value of μ for *sharp crested*

weirs. Hence for standard sharp crested weirs $\rho = 1$, while for all other crests, ρ becomes a multiplier, the value of which depends entirely on the nature of the crest.

The constants of Eq. (66) were derived from the values $\frac{2}{3}\mu$ found from the tabulated experiments 1 to 22, Table II. When inserted in Eq. (66) and calling $\rho = 1$, because all these experiments were made on sharp crested weirs, then for dimensions in feet:

$$\frac{2}{3}\mu_s = 0.40105 - 0.00453\left(\frac{H}{H+k}\right) + \frac{0.00348}{H} + 0.000132\, b \quad (67)$$

and for dimensions in meters:

$$\frac{2}{3}\mu_s = 0.40105 - 0.00453\left(\frac{H}{H+k}\right) + \frac{0.00106}{H} + 0.00043\, b \quad (68)$$

The values of $\frac{2}{3}\mu$ were then computed from Eq. (67) for each of the tabulations 1 to 32. The resulting errors, expressed in percentage, were entered in the last column of Table II.

The tabulations 41–45, Table III, were similarly treated and comparatively close agreements were found even though the experiments of Boileau were not conducted with the same degree of accuracy, nor under the same conditions as those in Table II.

The tabulations 33 to 40, in Table III, do not fit Eq. (67), because the values of k and H, and in fact all of the dimensions, are too small to promise any results which are comparable with those of Table II. However, the values of $\frac{2}{3}\mu_s$ were found from the new formulæ (22),* and for these values Eq. (66) becomes for metric units:

$$\tfrac{2}{3}\mu_s = 0.39085 + 0.14069\left(\frac{H}{H+k}\right) - 0.8738\, H + 0.00048\, b \quad (69a)$$

when the observations 33 to 37 only are included. For the remaining three observations 38 to 40 the following equation was found:

$$\tfrac{2}{3}\mu_s = 0.1843 + 0.3182\left(\frac{H}{H+k}\right) - 0.63\, H + 0.00048\, b\ . \quad (69b)$$

* The flume in these experiments widened out suddenly just beyond the discharge section, hence $(b + 0.042\, H)$ was used in place of b in Eq. (22) as was originally suggested by Francis.

TABLE II.

DETERMINATION OF μ FOR EQS. (19) TO (22) WHEN B=b. AND ρ=1.

No.	Original Experiment No.	Measured Values. B ft.	b ft.	k ft.	H ft.	v ft.	Q cu. ft.	Computed from Eqs. (22) $\frac{2}{3}\mu_s$	μ_s	Error by Eq. (67) Per cent.
	FROM TABLE XIII, EXPERIMENTS BY J. B. FRANCIS, 1852.									
1	67–71	9.992	9.995	5.048	0.79518	0.4071	23.7905	0.4068	0.6102	−0.2
2	44–50	9.992	9.995	5.048	0.97900	0.5403	32.5616	0.4052	0.6078	0.0
3	51–55	9.992	9.995	5.048	1.00026	0.5538	33.4946	0.4051	0.6077	0.0
	FROM TABLE XXVIII, EXPERIMENTS BY FTELEY AND STEARNS, 1878.									
4	1 & 5	5.0	5.0048	3.56	0.1509	0.054	1.007	0.4246	0.6369	0.0
5	6 " 10	5.0	5.0044	3.56	0.23035	0.098	1.8685	0.4164	0.6246	0.0
6	11 " 17	5.0	5.0045	3.56	0.33685	0.168	3.284	0.4121	0.6181	−0.1
7	18 " 21	5.0	5.0043	3.56	0.42425	0.233	4.6365	0.4097	0.6145	−0.1
8	22 " 27	5.0	5.0049	3.56	0.4305	0.237	4.736	0.4096	0.6143	−0.1
9	28 " 34	5.0	5.0047	3.56	0.5116	0.301	6.134	0.4083	0.6124	+0.1
10	36	5.0	5.0046	3.56	0.5477	0.331	6.796	0.4076	0.6114	0.0
11	37, 41, 44	5.0	5.0040	3.56	0.60076	0.375	7.8093	0.4070	0.6105	0.0
12	46 & 47	5.0	5.0042	3.56	0.69245	0.455	9.677	0.4063	0.6095	−0.1
13	53	5.0	5.0038	3.56	0.8047	0.556	12.147	0.4052	0.6078	0.0
	FROM TABLE XV, EXPERIMENTS BY FTELEY AND STEARNS, 1879.									
14	10	19.0	18.997	6.55	0.4685	0.151	20.178	0.4081	0.6122	+0.6
15	9	19.0	18.997	6.55	0.6460	0.239	32.685	0.4066	0.6099	+0.4
16	8	19.0	18.997	6.55	0.8191	0.334	46.760	0.4058	0.6087	+0.3
17	7	19.0	18.997	6.55	0.9853	0.433	62.023	0.4065	0.6097	0.0
18	6	19.0	18.997	6.55	0.9873	0.433	62.061	0.4055	0.6083	−0.2
19	5	19.0	18.997	6.55	1.1456	0.532	77.783	0.4052	0.6079	+0.2
20	3	19.0	18.997	6.55	1.2981	0.632	94.192	0.4055	0.6082	0.0
21	2	19.0	18.997	6.55	1.4546	0.737	112.066	0.4054	0.6081	0.0
22	1	19.0	18.997	6.55	1.6038	0.840	130.117	0.4053	0.6080	0.0
	FROM TABLE XIV, EXPERIMENTS BY FTELEY AND STEARNS, 1877.									
23	30	5.0	4.996	3.17	0.0746	0.023	0.3652	0.4450	0.6675	+1.0
24	29	5.0	4.996	3.17	0.0991	0.034	0.5498	0.4370	0.6555	+0.1
25	24	5.0	4.998	3.17	0.1225	0.046	0.7526	0.4345	0.6518	−0.9
26	20, 21	5.0	4.9965	3.17	0.16385	0.069	1.1536	0.4287	0.6431	−1.3
27	17, 19	5.0	5.0	3.17	0.21826	0.1033	1.75073	0.4219	0.6328	−1.0
28	14, 15	5.0	4.999	3.17	0.25325	0.1265	2.15975	0.4168	0.6252	−0.4
29	12, 13	5.0	4.996	3.17	0.32605	0.180	3.14475	0.4143	0.6215	−0.6
30	6, 8, 9	5.0	4.998	3.17	0.48443	0.3117	5.696	0.4112	0.6168	−0.7
31	4, 5	5.0	4.999	3.17	0.6737	0.488	9.376	0.4092	0.6138	−0.7
32	3	5.0	5.0	3.17	0.8118	0.627	12.466	0.4088	0.6132	−0.9

NOTE. — All above weirs were normal to the flow, had vertical faces making $\psi = 90°$ and had sharp crests consisting of a vertical, 1 inch planed, steel plate with beveled edge down stream. The above quantities represent means of the several observations bearing the observer's numbers in column 2.

TABLE III.

DETERMINATION OF μ_s FOR EQS. (19) TO (22) WHEN B=b.

No.	Original Experiment No.	Measured Values.						Computed From Eq. (22)		Error by Eq. (69) Per cent.
		B	b	k	H	$\frac{H}{k+H}$	Q	$\frac{2}{3}\mu_s$	μ_s	
		m.	m.	m.	m.		cu. m.			
FROM TABLE XXIII, EXPERIMENTS BY LESBROS, 1829–34. $\psi=90°$.										
33	$19\frac{52}{54}$	0.202	0.202	0.043	0.0955	0.68953	0.012905	0.4045	0.6067	0.0
34	$19\frac{55}{57}$	0.202	0.202	0.048	0.0955	0.66551	0.012561	0.3987	0.5981	+0.6
35	$19\frac{58}{60}$	0.202	0.202	0.070	0.088	0.55696	0.010532	0.3924	0.5886	0.0
36	$19\frac{61}{63}$	0.202	0.202	0.100	0.0805	0.44598	0.008638	0.3813	0.5720	+0.5
37	$19\frac{64}{66}$	0.202	0.202	0.130	0.0705	0.35162	0.006864	0.3788	0.5682	0.0
38	$19\frac{67}{69}$	0.202	0.220	0.020	0.0432	0.68354	0.003486	0.3747	0.5062	0.0
39	$19\frac{70}{73}$	0.202	0.202	0.030	0.0378	0.55752	0.00244	0.3380	0.5069	0.0
40	$19\frac{74}{77}$	0.202	0.202	0.050	0.0228	0.31319	0.000843	0.2653	0.3979	+1.6
FROM TABLE IX, EXPERIMENTS BY BOILEAU, 1845. $\psi=90°$.										
										Eq. 68.
41	—	0.895	0.895	0.340	0.0577	0.14508	0.0226025	0.4031	0.6046	−3.9
42	—	0.895	0.895	0.340	0.134	0.2827	0.0823787	0.4008	0.6012	−1.9
43	—	0.895	0.895	0.340	0.219	0.3918	0.17702	0.4029	0.6042	−0.6
44	—	1.616	1.616	0.468	0.0937	0.1668	0.0861429	0.4070	0.6105	−1.3
45	—	1.616	1.616	0.468	0.110	0.1903	0.108462	0.4013	0.6020	−2.3

FROM EXPERIMENTS BY J. B. FRANCIS ON AN OVERFALL WITH CREST 2.95 FEET WIDE AND FACE INCLINED. $\psi=20°$. DIMENSIONS IN FEET.

		B	b	k	H	v	Q	$\frac{2}{3}\mu$	$\frac{2}{3}\mu_s$	ρ
46	89	9.995	9.995	5.048	0.5872	0.238	13.385	0.3610	0.4078	0.8852
47	90	9.995	9.995	5.048	0.7904	0.358	20.892	0.3581	0.4062	0.8816
48	91	9.995	9.995	5.048	0.9767	0.480	28.914	0.3584	0.4052	0.8845
49	92	9.995	9.955	5.048	1.3252	0.725	46.183	0.3583	0.4041	0.8866
50	93	9.995	9.995	5.048	1.6338	0.963	64.346	0.3609	0.4034	0.8946

NOTE. — Experiments 41 to 45 were on sharp crested weirs and belong in all respects to the class of experiments in Table II, though made in metric measures.

The experiments 33 to 40, while in all other respects like those in Table II, were made on very small flumes and for weirs of very small heights k, for which Eqs. (67) and (68) did not apply.

Experiments 46 to 50 were made on a wide crested weir and with $\psi = 20°$ and $\phi = 90°$ otherwise the weir dimensions are like those of experiments 1 to 3. Hence this last set was used to show the effect of wide crests and the values $\frac{2}{3}\mu_s$ were found from Eq. (67) for sharp crested weirs, while the values $\frac{2}{3}\mu$ were computed from the experiments, using Eq. (20). The coefficients ρ were then obtained from μ_s and μ, and the values here found should be compared with those given at the end of this chapter.

To reduce Eq. (69) to feet units, the two last terms in each equation must be divided by 3.281.

A few experiments on a wide crested weir were made by Francis on the same flume used for the experiments 1 to 3, and this furnished a means of determining ρ for this particular case. The results are given in Table III, No. 46 to 50, see also note at foot of this table.

The subject of weir crests and their effect on discharge is of such vital importance that it will be treated separately; suffice it to say here, that the discharge for different crests may vary 20 per cent either way from the values obtained for standard sharp crested weirs.

The following Table IV contains results taken from the Cornell University experiments, made for the United States Board of Engineers on Deep Waterways, in June, 1899, by Mr. George W. Rafter.

These experiments cover a wide range of head H, though $B = b$ and k are constant throughout. The values $\frac{2}{3}\mu_s$, determined from the new formulæ (22), are seen to be uniformly decreasing for increasing values of H, while Mr. Rafter's coefficients, obtained from Bazin's formula $Q = mbH\sqrt{2\,gH}$, show no regular law.

The new coefficients thus found were used to determine the constants in Eq. (66) for $\rho = 1$, and the following equation was thus obtained for dimensions in feet:

$$\frac{2}{3}\mu_s = 0.3851 - 0.0258\frac{H}{H+k} + \frac{0.024}{H} + 0.000132\,b \quad . \quad (70a)$$

and for dimensions in meters:

$$\frac{2}{3}\mu_s = 0.3851 - 0.0258\frac{H}{H+k} + \frac{0.00731}{H} + 0.00043\,b \quad . \quad (70b)$$

A careful examination of the results in Table IV will show a very close agreement with the coefficients obtained in the previous and it should be noted that the same law of increase in μ_s for decrease in H is clearly demonstrated in all the experiments here given.

TABLE IV.

CORNELL UNIVERSITY EXPERIMENTS NOS. 20 AND 21, JUNE 1899, G. W. RAFTER. STANDARD SHARP CRESTED WEIRS WITH FREE OVERFALL.

No.	Measured Values.					Computed from Eqs. (22).		Computed from Eq. (70) $\frac{2}{3}\mu_s$	Rafter's Coef. m.
	B=b ft.	k ft.	H ft.	v ft.	Q cu. ft.	$\frac{2}{3}\mu_s$	μ_s		
1	6.53	5.26	0.7	0.327	12.7335	0.4066	0.5421	0.4171	0.4204
2	"	"	1.0	0.527	21.8755	0.4060	0.5413	0.4058	0.4174
3	"	"	1.5	0.902	39.8333	0.3981	0.5308	0.3962	0.4136
4	"	"	2.0	1.283	60.8596	0.3912	0.5216	0.3909	0.4106
5	"	"	2.5	1.679	85.0859	0.3879	0.5172	0.3873	0.4094
6	"	"	3.0	2.062	111.2059	0.3831	0.5108	0.3846	0.4094
7	"	"	3.5	2.459	140.6562	0.3819	0.5092	0.3825	0.4099
8	"	"	4.0	2.851	172.3920	0.3806	0.5075	0.3808	0.4112
9	"	"	4.5	3.240	206.4786	0.3798	0.5064	0.3794	0.4125
10	"	"	5.0	3.615	242.1977	0.3784	0.5045	0.3782	0.4133
11	"	"	5.5	3.979	279.5493	0.3761	0.5015	0.3771	0.4135
12	"	"	6.0	4.335	318.7293	0.3756	0.5008	0.3762	0.4136

NOTE. — The above experiments were made on a standard sharp crested weir with $\phi = \psi = 90°$ and $\rho = 1$.

(b.) *Coefficients* μ_s *for Eqs.* (19) *to* (22) *for weirs, normal to the channel but contracted by wing walls, for which* $B > b$.

The experiments given in Table V were used in Eqs. (21) to find $\frac{2}{3}\mu_s$ for the cases when $B > b$, and the values thus found were entered in columns 8 and 9.

The original equation (66) must now be modified to include variations between B and b, and may be expressed thus:

$$\frac{2}{3}\mu = \left[\alpha + \beta\left(\frac{bH}{B\,(H+k)}\right) + \frac{\gamma}{H} + \delta b\right]\rho \quad . \quad . \quad (71)$$

However, it was found for the experiments available for this case, that this equation did not give more accurate results than when the factor of β was taken as b/B, and therefore this latter value was finally adopted.

TABLE V.

μ_s FOR EQS. (19) TO (21) WHEN B>b.

No.	Original Experiment No.	Measured Values. B ft.	b ft.	k ft.	H ft.	Q cu. ft.	Computed from Eqs. (21). $\frac{2}{3}\mu_s$	μ_s	Errors by Eq. (72) Per cent.
FROM TABLE XIII, EXPERIMENTS BY J. B. FRANCIS, 1852.									
51	72–78	13.96	9.997	5.048	0.62355	16.2148	0.4047	0.6072	−0.1
52	56–61	13.96	9.997	5.048	0.79899	23.4305	0.4014	0.6021	0.0
53	11–33	13.96	9.997	5.048	0.99732	32.5798	0.3992	0.5988	0.0
54	5–10	13.96	9.997	5.048	1.24757	45.5654	0.3979	0.5969	0.0
55	1–4	13.96	9.997	5.048	1.55079	62.6019	0.3929	0.5894	+1.0
56	79–84	13.96	9.997	2.014	0.64928	17.4428	0.4023	0.6034	+0.3
57	62–66	13.96	9.997	2.014	0.82624	25.0410	0.4000	0.6000	+0.2
58	36–43	13.96	9.997	2.014	1.05033	36.0017	0.3989	0.5983	0.0
FROM TABLE XXVIII, EXPERIMENTS BY FTELEY AND STEARNS, 1878.									
59	4	5.0	3.008	3.56	0.2155	1.007	0.4146	0.6220	+1.0
60	9	5.0	3.0081	3.56	0.3301	1.869	0.4062	0.6092	0.0
61	14	5.0	3.008	3.56	0.4843	3.284	0.3990	0.5985	−0.1
62	31	5.0	3.007	3.56	0.7398	6.134	0.3930	0.5894	0.0
63	40 & 45	5.0	3.0101	3.56	0.8708	7.8075	0.3904	0.5856	+0.2
64	16	5.0	2.3132	3.56	0.5824	3.284	0.3941	0.5911	−0.5
65	24	5.0	2.3125	3.56	0.7478	4.736	0.3899	0.5848	−0.2
66	35	5.0	2.3126	3.56	0.9548	6.796	0.3867	0.5801	0.0
67	19	5.0	4.0058	3.56	0.4978	4.636	0.4038	0.6057	0.0
68	38 & 39	5.0	4.007	3.56	0.70595	7.815	0.3974	0.5916	+0.4
69	20	5.0	3.3107	3.56	0.5678	4.637	0.4018	0.6027	−0.9
70	32	5.0	3.3101	3.56	0.6860	6.134	0.3993	0.5989	−0.9
71	48 & 49	5.0	3.3095	3.56	0.9307	9.6485	0.3956	0.5934	−0.7

NOTE.—All of the above experiments were made on sharp crested weirs placed normal to the flow and vertical, so that $\phi = \psi = 90°$. While B > b, this difference was equally divided on both sides for all experiments 51 to 66, though for the five last lines, 67 to 71, the channel contraction was all on one side of the flume.

The constants were thus determined from the experimental values 51 to 66 and furnished the following for dimensions in feet:

$$\frac{2}{3}\mu_s = 0.3655 + 0.02357\left(\frac{b}{B}\right) + \frac{0.007328}{H} + 0.00093\, b \quad . \quad (72)$$

and for dimensions in meters:

$$\frac{2}{3}\mu_s = 0.3655 + 0.02357\left(\frac{b}{B}\right) + \frac{0.002384}{H} + 0.00305\, b \quad . \quad (73)$$

In the same table all the values for $\frac{2}{3}\mu_s$ were recomputed by Eq. (72) and the resulting percentage errors entered in the last column. This gave very concordant results.

However, for weirs in which k becomes an important factor, it may eventually be necessary to employ the more complicated form (71).

For weirs built diagonally to the flow, the above coefficients must necessarily answer the purpose until new experiments on such weirs shall have been made.

3. Incomplete Overfalls.

Very few experiments have ever been made on incomplete overfalls. In these the discharge takes place partly through a submerged area and partly as into free air, and coefficients μ_1 and μ have been assigned to the two discharges respectively.

Strangely enough, the submerged section has usually received

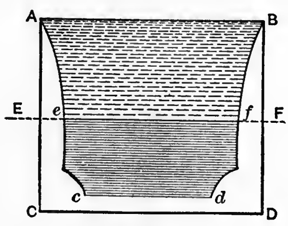


Fig. 31

the larger μ and as this seems contrary to all reason the following explanation is offered.

Let Fig. 31 represent the discharge section in which $\overline{ABCD}$ is the actual opening and $\overline{AecdfB}$ is the contracted discharge section

as for discharge into free air, making $\mu = 0.72$. Now if the lower portion of the section, $\overline{EFCD}$, is submerged, then most of the contraction belongs to this portion, while the upper area $\overline{AEFB}$ includes little of the contracted area. If these areas retain anything like this relation after the lower portion of the section is submerged, then certainly the ratios which the contracted areas bear to the respective flow areas must be less for the submerged portion. Hence it will be natural to assume $\mu_1 < \mu$, and as there is great liability of meeting with unforeseen influences on a partially submerged section, the former values μ_s for complete overfalls cannot safely be employed here.

The experiments of Francis, given in Table VI, offer a valuable contribution to the present subject and from them values of μ_1 and μ were determined for Eqs. (28).

The final equation (28) when solved for μ_1 or μ gives

$$\frac{2}{3}\mu = \frac{Q - \mu_1 b\left[H_1 - \dfrac{nV^2}{2\,g}\right]\sqrt{g\,(S_1 + S_2)}}{b\sqrt{2\,g}\,(S_1^{\frac{3}{2}} - S^{\frac{3}{2}})} \quad . \quad . \quad (74a)$$

$$\mu_1 = \frac{Q - \dfrac{2}{3}\mu b\sqrt{2\,g}\,(S_1^{\frac{3}{2}} - S^{\frac{3}{2}})}{b\left[H_1 - \dfrac{nV^2}{2\,g}\right]\sqrt{g\,(S_1 + S_2)}} \quad . \quad . \quad . \quad . \quad (74b)$$

either of which may be used when all the other terms are determined from experimental data, but a single set of observations will not be sufficient to solve one equation with two unknowns.

The following process was employed for the determination of μ_1 and μ.

Values were substituted in Eq. (74a) for two sets of observations having approximately the same dimensions for H_2. Now from the behavior of $\frac{2}{3}\mu$ for complete overfalls, it is found that for similar values of H_2, $\frac{2}{3}\mu$ is practically the same. Hence, each such pair of equations may be equated so as to involve only μ_1, which would then represent the mean value for the two sets of observations. Then using this value of μ_1 in each of the pairs of equations, the mean value $\frac{2}{3}\mu$ was found.

TABLE VI.

DETERMINATION OF μ FOR EQS. (28), $B = b$, $\rho = 1$.

EXPERIMENTS BY J. B. FRANCIS, 1883.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	16	20	21
OBS	Measured Values.									Computed.		By Eq. (67) μ_s	Computed By Eqs. (74).		Diff's. Betw. the μ's.		Ratio. $\frac{H_1}{H}$	Computed Q.		Diff. in Q Col. 20–8 %
No.	B	b	k	H	H_1	H_2	Q	B_1	k_1	v	V		μ	μ_1	μ and μ_s %	μ_1 and μ %		Francis' Eq. (75) cu. ft.	Eqs. (28) (76) (77) cu. ft.	
	ft.	ft.	ft.	ft.	ft.	ft.	cu. ft.	ft.	ft.	ft.	ft.									
25	11.22 } 10.98 }	11.22 } 10.98 }	5.7	1.491	1.039	0.452	94.16	32.2	7.3	0.5898	0.3506	0.6080	0.5366	0.5762	−11.7	+7.3	0.6968	93.77	94.16	0.0
95	22.2	22.2	5.7	1.156	0.657	0.499	73.15	32.2	7.3	0.4806	0.2855	0.6093	0.5726	0.5723	−6.2	−0.05	0.5683	73.44	73.21	+0.07
96	22.2	22.2	5.7	1.149	0.636	0.513	73.04	32.2	7.3	0.4804	0.2858	0.6093	0.5879	0.5616	−3.5	−4.5	0.5535	73.67	73.04	0.0
94	22.2	22.2	5.7	1.091	0.448	0.643	73.37	32.2	7.3	0.4867	0.2941	0.6097	0.6225	0.5257	+2.1	−15.5	0.4106	74.80	73.85	+0.6
93	22.2	22.2	5.7	1.037	0.263	0.774	73.48	32.2	7.3	0.4913	0.3017	0.6099	0.6211	0.5276	+1.8	−15.0	0.2536	73.96	73.66	+0.2
22	22.2	22.2	5.7	1.227	0.309	0.918	95.00	32.2	7.3	0.6178	0.3877	0.6090	0.6200	0.5299	+1.8	−14.5	0.2518	95.25	94.91	−0.1
21	22.2	22.2	5.7	1.203	0.207	0.996	95.12	32.2	7.3	0.6589	0.3935	0.6090	0.6196	0.5292	+1.7	−14.4	0.1721	94.57	94.94	−0.2
73	22.2	22.2	5.7	2.190	1.071	1.119	202.54	32.2	7.3	1.1563	0.7514	0.6065	0.6190	0.5298	+2.0	−14.4	0.4890	203.17	201.98	−0.3
81	22.2	22.2	5.7	2.319	1.111	1.208	224.17	32.2	7.3	1.2592	0.8277	0.6063	0.6151	0.5371	+1.4	−12.7	0.4791	222.81	223.34	−0.3
43	22.2	22.2	5.7	1.720	0.466	1.254	159.25	32.2	7.3	0.9668	0.6368	0.6074	0.6149	0.5384	+1.2	−12.4	0.2709	157.09	158.57	−0.4
69	22.2	22.2	5.7	2.034	0.528	1.506	203.01	32.2	7.3	1.1824	0.8054	0.6067	0.6083	0.5342	+0.26	−12.2	0.2595	202.78	203.76	+0.4
68	22.2	22.2	5.7	1.994	0.327	1.667	203.16	32.2	7.3	1.1894	0.8272	0.6068	0.6078	0.5350	+0.16	−12.0	0.1640	202.21	203.77	+0.3

NOTE. — B_1 was the width of canal after passing the weir and k_1 was the downstream height of the weir, both used in computing V. The weir was the standard sharp crested, built normal to the flow, and ϕ and ψ were both 90°. The experiments are tabulated with respect to increasing values of H_2, while $\rho = 1$ and n in Eqs. (28) was taken equal to 0.67.

Now Eq. (67) permits of finding very close values for the differences between successive values $\frac{2}{3}\mu$ when the corresponding differences in H_2 are known. Hence the first values of $\frac{2}{3}\mu$, above found, were corrected for these small differences and the corrected values then used to determine the final values of μ_1 from Eq. (74*b*). These latter values are entered in columns 14 and 15 of Table VI.

Using these coefficients in Eqs. (28) the quantity was computed for each of the tabulated experiments and these results entered in column 20. The remarkably close agreement which these quantities bear to the measured values is shown by the percentage differences in column 21.

For the sake of comparison, the values μ_s from Eq. (67) were computed for each experiment and entered in column 13, and the percentage differences between these and the μ's in column 14 are given in column 16, while the percentage differences between each pair of μ_1 and μ are entered in column 15.

In column 19 the values of Q are given. These result from the Francis formula (47*a*) as follows:

$$Q = 3.33\, b\,(H - H_1)^{\frac{3}{2}} + 4.5988\, bH\sqrt{H - H_1}\,. \quad . \ (75)$$

The values $\dfrac{H_1}{H}$ were also computed as given in column 18, because the coefficient C in the formula of Messrs. Fteley and Stearns is made to depend on this ratio.

By careful inspection of Table VI the following conclusions may be drawn:

1. The ratio $\dfrac{H_1}{H}$ does not seem to have any definite relation to either of the coefficients μ except that for all ratios exceeding 0.5 the values μ are irregular.

2. This irregularity in the coefficients for the first three experiments, 25, 95 and 96, for which H_2 was small as against the submerged depth H_1, is probably due to the very large value of k compared with both H_1 and H_2. It is possible under these

extreme conditions that the impact, of the flow of approach against the weir, exerts an influence on the whole discharge area.

3. The values μ, in column 14, increase rapidly for the first four experiments and then slowly diminish with increasing values of H_2 as is indicated by the percentages in column 16.

4. The coefficients μ_1, in column 15, follow an exactly opposite law of increase, and from the fourth experiment down μ_1 is from 15.5 to 12 per cent less than the corresponding μ.

5. Francis observed the fact that for small values of H_1 (from 0.08 to 0.17 feet) the discharge over the crest takes place as for discharge into free air. That is, some air is still carried over the crest and no suction occurs on the discharge area. When H_1 becomes larger, this air is expelled and suction effect follows with increased discharge. Hence it is very important in conducting experiments of this kind to note these conditions carefully lest the entire work may prove useless.

6. The present experiments do not manifest the same accuracy as was observed for those dealing with complete overfalls. Also the quantity of discharge was not directly measured but computed from velocity experiments.

Hence there is still need for further experiments on incomplete overfalls under more varying conditions as when $B > b$, etc., and careful attention should be given to measure such dimensions as will make possible the separate determination of μ_1 and μ.

It was impossible to fit any formula to the values μ_1 and μ of the first three experiments. For heights H_2, between 0.643 and 1.119 feet, the following formulæ were adopted:

For dimensions in feet:

$$\left.\begin{aligned} \tfrac{2}{3}\,\mu &= 0.4001 + \frac{0.01038}{H_2} + 0.000146\,b \\ \mu_1 &= 0.5274 + 0.000146\,b, \end{aligned}\right\} \quad . \; . \; (76a)$$

and for dimensions in meters:

$$\left.\begin{aligned} \tfrac{2}{3}\,\mu &= 0.4001 + \frac{0.00316}{H_2} + 0.00048\,b \\ \mu_1 &= 0.5274 + 0.00048\,b. \end{aligned}\right\} \quad . \; . \; (76b)$$

When the pressure height H_2 becomes larger, the following formulæ are preferable:

For dimensions in feet:

$$\left.\begin{aligned} \tfrac{2}{3}\,\mu &= 0.4001 + \frac{0.00799}{H_2} + 0.000146\,b \\ \mu_1 &= 0.5346 + 0.000146\,b, \end{aligned}\right\} \quad . \quad . \quad (77a)$$

and for dimensions in meters:

$$\left.\begin{aligned} \tfrac{2}{3}\,\mu &= 0.4001 + \frac{0.00244}{H_2} + 0.00048\,b \\ \mu_1 &= 0.5346 + 0.00048\,b. \end{aligned}\right\} \quad . \quad . \quad (77b)$$

All the above formulæ (76) and (77) are for sharp crested weirs for which $\rho = 1$. For any other kind of crest the values for ρ must be determined from experiments still remaining to be made.

4. Sluice Weirs and Gates.

Among the available experiments which can be used for the determination of μ in Eqs. (36), there are only a few which are entitled to any confidence. Those given in Table VII comprise about all for which sufficient data were given to permit of their use.

The largest experiments of this class are those made on the Danube Canal, 1876 to 1883, by Freiherr *v.* Engerth. These three experiments are tabulated as Nos. 1, 2 and 3 and the μ_1 were found by inserting the observed values in the new formulæ (36), using $n = 0.67$.

For further values μ_1 the experiments given in the table were used. These were all made prior to 1880, by the following: No. 4–11 by Lesbros; No. 12–22 by Boileau; No. 23–34 by Weisbach; and No. 35–41 by Bornemann. The computation of values μ_1 for these experiments in Eqs. (36) resulted in figures entered in the table, and the percentage differences between these and the values found from Eqs. (78) to (81) are given in the last column.

TABLE VII.

DETERMINATION OF μ_1 AND μ FOR EQS. (36) TO (39).

Experiment by	No.	Original No.	Measured Values. All in Meters. B	b	a	k	H	T_1	H_2	Q	Observer's Values. μ	From Eqs. (36) (38) (39) μ and μ_1	Diff. Betw. μ and Values by Eqs. (78) to (82) %	
Engerth.	1	22./2. 1876	50.0	46.58	3.15	0.0	8.07	7.10	0.97	602.9	0.664	0.808	0.0	μ_1 Eq. (78)
	2	14./3. *1881	50.0*	46.58	2.32	0.0	7.51	6.37	1.14	477.3	0.686	0.835	+2.7	
	3	5./1. 1883	50.0	46.58	3.25	0.0	8.72	7.28	1.44	743.3	0.677	0.815	+0.3	
Lesbros.	4	19	3.68	0.6	0.03	0.54	...	...	0.255	0.027116	0.6943	0.6935	+0.1	μ Eq. (80)
	5	16	3.68	0.6	0.03	0.54	...	...	1.770	0.071377	0.6759	0.6755	+3.1	
	6	13	3.68	0.6	0.05	0.54	...	...	0.501	0.062359	0.6803	0.6794	−1.6	
	7	11	3.68	0.6	0.05	0.54	...	...	1.870	0.121188	0.6715	0.6711	−0.1	
	8	8	3.68	0.6	0.20	0.54	...	...	0.681	0.259513	0.6406	0.6388	−1.6	
	9	6	3.68	0.6	0.20	0.54	...	...	1.872	0.450291	0.6367	0.6355	−2.0	
	10	3	3.68	0.6	0.40	0.54	...	...	1.037	0.600745	0.6178	0.6159	0.0	
	11	1	3.68	0.6	0.40	0.54	...	...	1.8903	0.823527	0.5983	0.5945	+2.7	
Boileau.	12	3	0.9	0.9	0.0485	0.0	...	...	0.1965	0.047955	0.591	0.582	+2.2	μ Eq. (81)
	13	1	0.9	0.9	0.0485	0.0	...	...	0.503	0.080237	0.599	0.592	+1.1	
	14	5	0.898	0.898	0.080	0.0	...	...	0.400	0.112683	0.586	0.579	+1.1	
	15	4	0.898	0.898	0.080	0.0	...	...	0.541	0.137777	0.609	0.602	−2.6	
	16	10	0.9	0.9	0.0994	0.0	...	...	0.1435	0.075826	0.573	0.568	+0.4	
	17	7	0.9	0.9	0.0994	0.0	...	...	0.365	0.13348	0.592	0.583	−0.6	
	18	6	0.9	0.9	0.0997	0.0	...	...	0.630	0.181700	0.598	0.591	−1.4	
	19	11	0.898	0.898	0.120	0.0	...	...	0.444	0.17544	0.586	0.577	0.0	
	20	—	0.90	0.90	0.0997	0.0	0.5890	0.4905	0.0985	0.086043	0.685	0.665	—	μ_1
	21	—*	0.90	0.90	0.0997	0.0	0.6295	0.5115	0.1180	0.093026	0.677	0.664	—	
	22	—	0.90	0.90	0.0997	0.0	0.5450	0.4235	0.1215	0.099963	0.715	0.696	—	

Weisbach.	23	1	0.36395	0.36395	0.0589	0.0	...	...	0.1495	0.02607	0.756	0.733	−0.5	μ Eq. (82)
	24	5	0.36395	0.36395	0.0589	0.0	...	...	0.2901	0.03898	0.776	0.775	+0.4	
	25	9	0.364	0.364	0.0327	0.0	...	...	0.137	0.01542	0.824	0.802	−0.5	
	26	6	0.364	0.364	0.0327	0.0	...	...	0.2916	0.02370	0.853	0.838	−0.2	
	27	14	0.3635	0.3635	0.104	0.0	...	...	0.2004	0.04915	0.709	0.690	−1.0	
	28	13	0.3635	0.3635	0.104	0.0	...	...	0.2812	0.06032	0.725	0.705	+1.7	
	29	3	0.363	0.3632	0.059	0.0	0.2202	0.1640	0.0562	0.017424	0.758	0.721	—	μ_1
	30	8	0.363	0.3632	0.059	0.0	0.2071	0.1468	0.0603	0.018995	0.794	0.746	—	
	31	4*	0.363	0.3632	0.059	0.0	0.2816	0.1916	0.0900	0.020582	0.715	0.689	—	
	32	10	0.363	0.362	0.07725	0.0	0.2713	0.1931	0.0782	0.030117	0.844	0.787	—	
	33	9*	0.363	0.362	0.07725	0.0	0.3061	0.2266	0.0795	0.026358	0.741	0.709	—	
	34	12	0.363	0.362	0.07725	0.0	0.2758	0.1840	0.0918	0.033476	0.866	0.802	—	
Bornemann.	35	11	0.544	0.520	0.204	0.0	0.356	0.342	0.014	0.05247	0.842	0.756	+3.4	μ_1 Eq. (79)
	36	16	0.544	0.520	0.158	0.0	0.394	0.355	0.039	0.05247	0.705	0.667	+7.3	
	37	9	0.544	0.520	0.127	0.0	0.379	0.290	0.089	0.07259	0.813	0.752	−2.1	
	38	5	0.544	0.520	0.091	0.0	0.447	0.260	0.187	0.07259	0.794	0.757	−6.1	
	39	17	0.802	0.774	0.131	0.0	0.274	0.263	0.011	0.04779	0.919	0.827	−6.0	
	40	7	0.802	0.774	0.110	0.0	0.305	0.274	0.031	0.0488	0.712	0.677	−7.2	
	41	2	0.802	0.774	0.051	0.0	0.356	0.204	0.152	0.0488	0.713	0.695	−0.7	

NOTE: In the above $n = 0.67, \rho = 1, \phi = 90°$ and $\varphi = 90°$.

Experiments 1:3 were made for entirely submerged discharge, Fig. 19.
" 4:11 " " " free discharge with complete contraction.
" 12:19 " " " free discharge with *no contraction* on sides and bottom.
" 20:22 " " " entirely submerged discharge no contraction on sides and bottom.
" 23:28 " " " free discharge through sluice gate with rounded edge and no contraction on sides and bottom.
" 29:34 for submerged discharge through sluice gates without side or bottom contraction.
" 35:41 for submerged discharge through sluices without bottom contraction.

* These experiments are probably in error.

The coefficients μ, which the various observers found from the use of their own formulæ, are given in the 12th column, and the following comments regarding these may not be out of place here.

Excepting the first three cases, the general comparison shows the new values to be somewhat less than those given by the observers themselves, and this is undoubtedly so because the new formulæ take into account several conditions not considered by the older formulæ.

According to the experiments of Engerth, Boileau and Weisbach on sluices wherein the sill was level with the bottom of the flume or canal, or when $k = 0$, H_2 varies with μ_1, while according to Lesbros for $k = 0.54$ m. this relation is inverse. The only experiments which form exceptions to this rule are Nos. 2, 21, 31 and 33, for which reason it is fair to assume that some small errors were committed in those cases.

Weisbach's experiments, 23–28, were made on a sluice without side and bottom contractions and a rounded edge on the gate, which explains the rather high values there obtained for μ_1.

It should also be observed that for these experiments the coefficient μ_1, for submerged discharge, is smaller than for free discharge, which is exactly the opposite of our previous findings in this regard. However, the present case is different inasmuch as the contraction, whether for free or submerged discharge, takes place all around the opening when dealing with a sluice gate. Also the suction produced by the water leaving the discharge section, tends to increase the quantity for submerged discharge as against free discharge where no suction occurs. Therefore, these phenomena would indicate just what the values μ indicate to be the true condition.

Hence these tabulations of μ_1 and μ show quite conclusively that the new formulæ apply to a rather wide range of conditions and can, therefore, be expected to give very much greater accuracy than was possible to attain with the older formulæ which did not include the many variable influences always attending hydraulic problems.

However, these experiments are of such nature that their practical applicability is very limited and until more extensive experiments of this kind shall have been made these values must be accepted as the best information available at this time.

The following formulæ for μ and μ_1 are given with reserve because the data on which they are based is entirely inadequate for the deduction of such formulæ. However, they are better than nothing and may occasionally serve a useful purpose in want of something better.

Bornemann gave a formula for μ_1 which seems to give fairly accurate results. This formula with slightly modified coefficients and another member involving b was used.

For the first three experiments by Engerth, there is not enough data to justify any formula; however, the following one is offered for submerged discharge through large sluice openings:

For dimensions in feet:

$$\mu_1 = 0.7069 - \frac{0.2717\sqrt{a}}{H - \frac{a}{2}} + 0.00093\,b \quad . \quad . \quad . \quad (78a)$$

and for dimensions in meters:

$$\mu_1 = 0.7069 - \frac{0.15\sqrt{a}}{H - \frac{a}{2}} + 0.00305\,b \quad . \quad . \quad . \quad (78b)$$

The other experiments on submerged discharge are too unreliable and, therefore, formulas could not be found except for the last experiments 35–41, by Bornemann, for which the following are given:

For dimensions in feet:

$$\mu_1 = 0.4988 + \frac{0.271\sqrt{a}}{H - \frac{a}{2}} + 0.00093\,b \quad . \quad . \quad . \quad (79a)$$

and for dimensions in meters:

$$\mu_1 = 0.4988 + \frac{0.14965\sqrt{a}}{H - \frac{a}{2}} + 0.00305\,b \quad . \quad . \quad . \quad (79b)$$

The experiments 4–11 by Lesbros, for free discharge with complete contraction, give for dimensions in feet:

$$\mu = 0.5708 + \frac{0.01355\sqrt{a}}{\sqrt{H_2 + \frac{a}{2}}} + \frac{0.0382}{\sqrt{a}} + 0.00131\, b \quad . \quad (80a)$$

and for dimensions in meters:

$$\mu = 0.5708 + \frac{0.01355\sqrt{a}}{\sqrt{H_2 + \frac{a}{2}}} + \frac{0.02109}{\sqrt{a}} + 0.00431\, b \quad . \quad (80b)$$

For the experiments 12–19 by Boileau on free discharge through gates of same size as the flume, without bottom sills, thus allowing the discharge to proceed without side or bottom contraction, the formula becomes for dimensions in feet:

$$\mu = 0.5751 - \frac{0.01898\sqrt{a}}{\sqrt{H_2 - \frac{a}{2}}} + \frac{0.00492}{a} + 0.000146\, b \quad . \quad (81a)$$

and for dimensions in meters:

$$\mu = 0.5751 - \frac{0.01898\sqrt{a}}{\sqrt{H_2 - \frac{a}{2}}} + \frac{0.00144}{a} + 0.00048\, b \quad . \quad (81b)$$

For experiments 23–28 by Weisbach, for free discharge like experiments 12–19, without contraction but having the bottom edge of the gate rounded off, the following formula was found for dimensions in feet:

$$\mu = 0.8452 - \frac{0.21936\sqrt{a}}{\sqrt{H_2 - \frac{a}{2}}} + \frac{0.00718}{a} + 0.000146\, b \quad . \quad (82a)$$

and for dimensions in meters:

$$\mu = 0.8452 - \frac{0.21936\sqrt{a}}{\sqrt{H_2 - \frac{a}{2}}} + \frac{0.00219}{a} + 0.00048\, b \quad . \quad (82b)$$

5. End Contraction.

In all the new formulæ it is supposed that the discharge continues with a uniform width of section b which is true when the water, after leaving the weir crest, is confined by lateral walls. All of these cases are known as discharge *without end contraction,* the term *end* being applied to the ends of the weir adjacent to the sides of the canal.

When the discharge is not thus laterally confined and is allowed to take place into free air, then end contraction takes place and the quantity of discharge is slightly reduced.

The exact amount of reduction thus produced can only be estimated. Francis proposed a simple formula for this purpose in the form of a correction to the length b of the weir, by subtracting an amount 0.1 H for each such end contraction.

Thus the new $b = (b - 0.1\, nH)$ . . . . (83)

where n = number of end contractions.

Undoubtedly this covers the case in a very approximate manner but in the absence of other more extended experiments to determine the effect of end contractions on the discharge, the above advice must be followed.

6. Weir Crests.

Coefficient ρ for Different Styles of Crests, for Complete Overfalls.

The following data have been taken from pp. 71–75, "Hydraulic Tables" by Professor Gardner S. Williams and Mr. Allen Hazen, published by Messrs. John Wiley and Sons, in 1905. For the kind permission granted by these gentlemen, to use this very valuable collection of data, the author here repeats his expression of gratitude and thanks.

The following tables were derived from experiments conducted under the personal supervision of Professor Gardner S. Williams at the hydraulic laboratory of Cornell University between 1899 and 1904. Some of these experiments were made for the United States Geological Survey, some for the United States Board of Engineers

on Deep Waterways, and those on the Croton Dam model were made at the request of Mr. John R. Freeman. The figures, as here given, represent the results deduced by Professor Williams, and are entitled to the utmost confidence.

The triangular weirs with sloped face upstream, are omitted here because this effect is included in the new formulæ.

In general then, the coefficient ρ from Eq. (66) is found to be $\rho = \dfrac{\mu}{\mu_s}$, where μ_s is the empiric coefficient for discharge over standard sharp crested weirs and coefficient μ is the for any other crest. Hence, having found μ_s from the foregoing, the μ for any particular kind of weir crest, other than the standard, may be found from

$$\mu = \mu_s\rho, \text{ or } \tfrac{2}{3}\,\mu = \tfrac{2}{3}\,\mu_s\rho \quad . \quad . \quad . \quad . \quad . \quad (84)$$

This is equivalent to calling ρ a multiplier with which to multiply μ_s for standard sharp crested weirs, to find μ for any weir of exactly the same general dimensions and conditions of flow differing only in shape of the crest. Hence ρ may also be called the crest coefficient, which represents the change in the contraction of the discharge section due to the change in the crest from the standard sharp crest.

In order that ρ may be determined experimentally for different kinds of crests it is, therefore, necessary to make duplicate experiments first with the standard sharp crest and then with the experimental crest, all other conditions remaining exactly alike.

This was not done by Mr. Rafter for the very elaborate experiments made by him at Cornell University in 1899. There all the experiments for standard sharp crested weirs were confined to varying the head H, all dimensions of weir and channel remaining constant. The experiments on other forms were widely varied in height k and other dimensions and hence the values $\dfrac{m}{m'}$ expressing the relation intended to be expressed by ρ, did not contribute much of scientific value because the cases thus compared were not strictly comparable.

The following data from the above named "Hydraulic Tables" is believed to be the most accurate in existence at this time and aside from a few general remarks the tables are self explanatory.

All the formulæ previously presented do not include the element of weir crests and the above values for μ_s are for sharp crested weirs of standard type, so that the μ_s once found for a particular standard weir, the multiplier ρ to be used to find μ for any other than standard weir, is given below.

"On all the models having vertical downstream faces, including model *P*, air was admitted to the space underneath the sheet. On models *D* and *E*, experiments were made with the space underneath the sheet unaërated, so that a partial vacuum existed there, which is shown to increase the discharge about 5 per cent at the high heads. For the weirs with inclined downstream faces, models *F* to *O* inclusive, no air was admitted under the sheet. A comparison of the results upon models *G* and *H*, shows the effect of rounding the upstream corner of the weir to be an increase in discharge of about 4 per cent at the high heads."

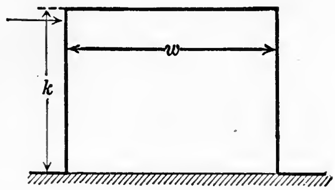


TABLE VIII. — RECTANGULAR FLAT CRESTED WEIRS.

VALUES OF ρ FOR SAME *b* AND *k*.

H Feet.	When w =							
	0.48 ft.	0.93 ft.	1.65 ft.	3.17 ft.	5.84 ft.	8.98 ft.	12.24 ft.	16.30 ft.
0.5	0.902	0.830	0.819	0.797	0.785	0.783	0.783	0.783
1.0	0.972	0.904	0.879	0.812	0.800	0.798	0.795	0.792
1.5	1.000	0.957	0.910	0.821	0.807	0.803	0.802	0.797
2.0	1.000	0.989	0.925	0.821	0.805	0.800	0.798	0.795
2.5	1.000	1.000	0.932	0.816	0.800	0.795	0.792	0.789
3.0	1.000	1.000	0.938	0.813	0.796	0.791	0.787	0.784
3.5	1.000	1.000	0.942	0.810	0.793	0.787	0.783	0.780
4.0	1.000	1.000	0.947	0.808	0.790	0.783	0.780	0.777

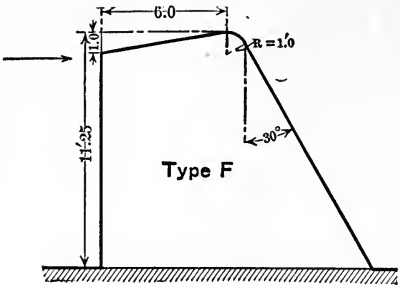
H = depth of water flowing over the crest of the weir.

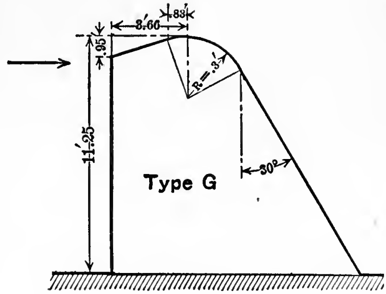
TABLE IX.—COMPOUND WEIRS.

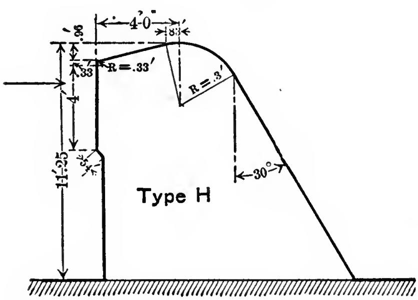
VALUES OF ρ FOR VARIOUS TYPES OF WEIRS F TO L.

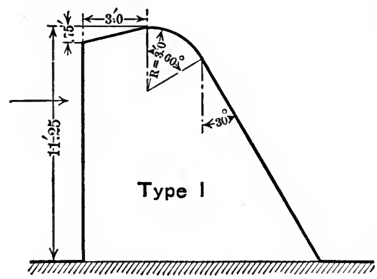
H Feet.	Type F.	Type G.	Type H.	Type I.	Type J.	Type K.	Type L.
0.5	0.964	0.932	0.934	0.968	0.971	0.971	0.971
1.0	1.026	0.982	1.000	1.008	1.040	1.040	0.983
1.5	1.064	1.015	1.040	1.032	1.083	1.092	1.012
2.0	1.066	1.031	1.061	1.041	1.105	1.126	1.040
2.5	1.025	1.038	1.073	1.043	1.118	1.146	1.057
3.0	0.992	1.044	1.082	1.044	1.128	1.163	1.072
3.5	0.966	1.049	1.090	1.045	1.136	1.177	1.085
4.0	0.944	1.053	1.097	1.046	1.144	1.190	1.097

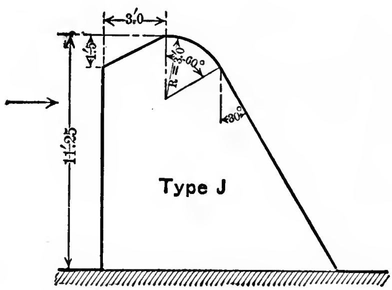
NOTE: See cuts on opposite page.

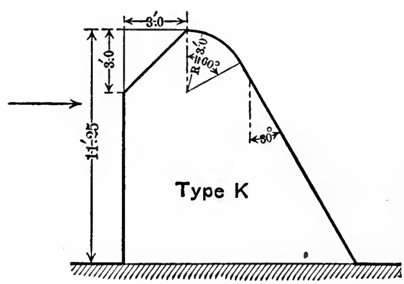

6.0
1.0
R = 1.0
11.25
30°
Type F

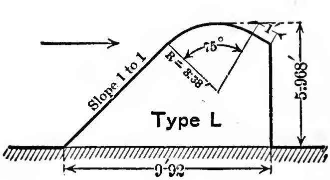

3.66
.83
.95
R = 3'
11.25
30°
Type G


4.0
.83
R = .33'
.33
R = 3'
4
11.25
30°
Type H


3.0
.75
R = 3.0
60°
30°
11.25
Type I


3.0
1.5
R = 3.0
60°
30°
11.25
Type J


3.0
3.0
R = 3.0
60°
30°
11.25
Type K


75°
R = 8.38
Slope 1 to 1
5.968
Type L
9.92

TRAPEZOIDAL WEIRS.

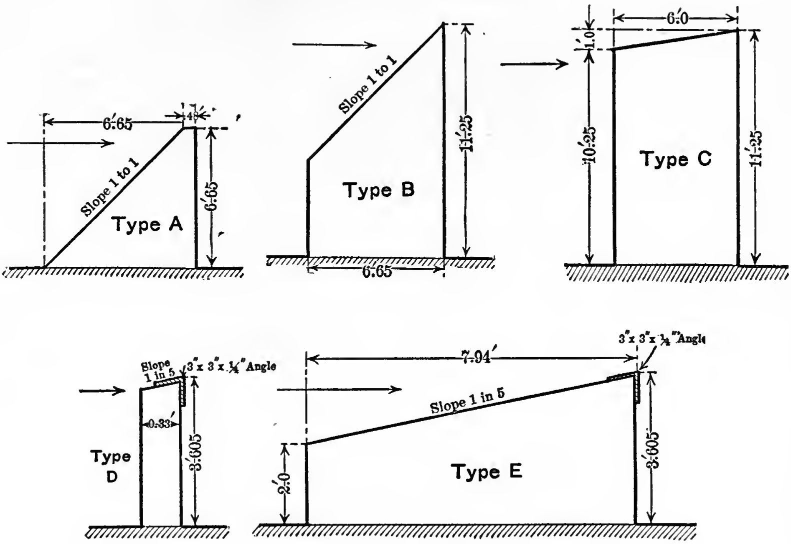


TABLE X.

VALUES OF ρ FOR TYPES A TO E FOR SAME b AND k.

H ft.	Type A.	Type B.	Type C.	Type D.	D with Vacuum.	Type E.	E with Vacuum.
0.5	0.968	1.060	1.043	1.069	1.088	1.069	1.069
1.0	1.071	1.079	1.040	1.079	1.106	1.079	1.079
1.5	1.077	1.091	1.037	1.084	1.117	1.088	1.092
2.0	1.081	1.096	1.027	1.057	1.092	1.063	1.083
2.5	1.077	1.093	1.015	1.041	1.079	1.049	1.081
3.0	1.074	1.090	1.005	1.028	1.068	1.039	1.080
3.5	1.071	1.087	0.996	1.018	1.059	1.029	1.079
4.0	1.069	1.085	0.989	1.009	1.051	1.021	1.078

COMPLEX WEIRS.

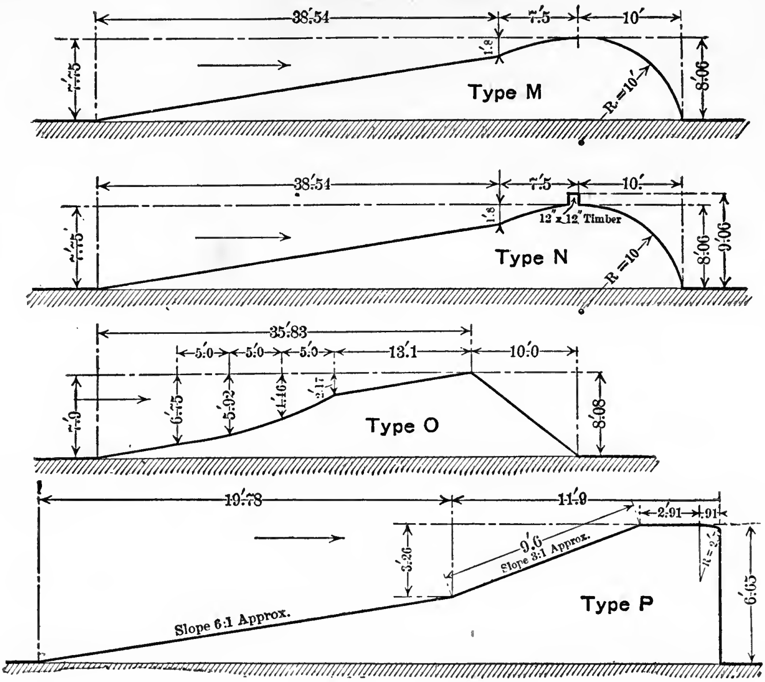


TABLE XI.

VALUES ρ FOR TYPES M TO P.

H ft.	Type M.	Type N.	Type O.	Type P.
0.5	0.964	0.897	1.095	0.920
1.0	0.965	0.946	1.088	0.915
1.5	0.963	0.999	1.084	0.914
2.0	0.949	1.025	1.069	0.935
2.5	0.933	1.039	1.051	0.950
3.0	0.920	1.052	1.035	0.962
3.5	0.911	1.063	1.024	0.972
4.0	0.903	1.072	1.014	0.982

APPENDIX A.

A COLLECTION OF WEIR FORMULÆ PROPOSED BY DIFFERENT AUTHORS, WITH DISCUSSION.

I. COMPLETE OVERFALLS.

THE following, Fig. I, shows the lettered dimensions used in the formulæ for complete overfalls.

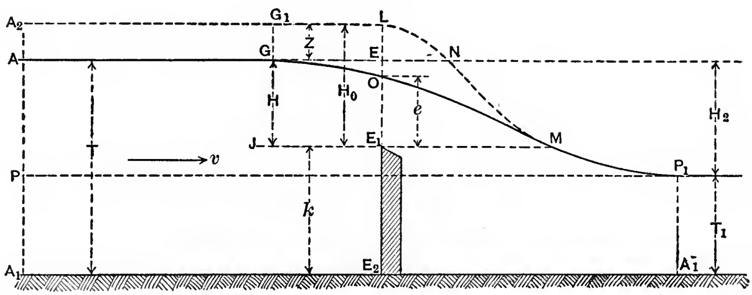


Fig. I

Eytelwein in his "Handbuch der Mechanik fester Koerper und der Hydraulik," 1823; also,

Weisbach in "Huelse's Maschinen-Encyclopædie," 1841, give the following formula:

$$Q = \frac{2}{3}\,\mu b\sqrt{2\,g}\left[\left(H + \frac{v^2}{2\,g}\right)^{\frac{3}{2}} - \left(\frac{v^2}{2\,g}\right)^{\frac{3}{2}}\right] \quad . \quad . \quad . \quad (1)$$

Let $\overline{A_2L}$ represent the level of a quiet reservoir such that $\overline{JG_1} = H + \frac{v^2}{2\,g} = H_0$.

Then according to Eq. (1), Chap. I, the discharge through an opening of height H_0 would be

$$Q_1 = \frac{2}{3}\mu b\sqrt{2g}\left(H + \frac{v^2}{2g}\right)^{\frac{3}{2}},$$

and taking off the small quantity passing through $\overline{LE}$ and which is

$$q = \frac{2}{3}\mu b\sqrt{2g}\left(\frac{v^2}{2g}\right)^{\frac{3}{2}}$$

the final discharge through $\overline{E_1E}$ becomes

$$Q = Q_1 - q = \text{the above formula (1).}$$

The Eytelwein-Weisbach formula is thus derived from the fundamental Eq. (1), for flow through a lateral orifice, and, therefore, does not apply to discharge with initial velocity of approach.

The four following proofs of the incorrectness of the Eytelwein-Weisbach formula are now presented.

1. No consideration is given to width and depth of the approach channel; to the height k of the weir, nor to the shape of the weir. All of these circumstances are known to exert a very considerable influence on the discharge.

2. The flow of approach is supposed to exert an hydrodynamic pressure of $\frac{v^2}{2g}$ making the total hydraulic pressure on the discharge section $\left(H + \frac{v^2}{2g}\right)$. It was shown, however, that this depends on H, b, B, and k, and may have any value between $4\left(\frac{v^2}{2g}\right)$ and $15\left(\frac{v^2}{2g}\right)$.

3. These authors were undoubtedly cognizant of some of these defects in their formula and sought to correct the error by the coefficient μ. However, a rational formula must express the true influence of all the hydrostatic and hydrodynamic pressures and

weir dimensions on the discharge. The coefficient μ should serve merely to rectify the theoretically correct discharge to cover the unknowable effects due to adhesion, cohesion, friction and contraction.

4. The values of μ, being variable in character, are necessarily without value except within the scope of the experiments from which they were obtained. For rational formulæ this should not be the case, at least not to any great extent. This becomes the more important when it is realized that all hydraulic experiments must be confined to reasonably small conditions, and unless a rational solution of hydraulic problems is made possible the ultimate solution applicable to extensive waterpower plants would remain impossible.

The above Eytelwein-Weisbach formula was discussed at some length because it is in most general use and also because the objections cited will be found equally applicable to most other formulæ.

Navier proposed the following formula, wherein the head $\overline{EO} = 0.2753\ H$ (see Fig. 1), a relation based on the doctrine of least work. The formula is

$$Q = \tfrac{2}{3}\ \mu b \sqrt{2\,g}\,[1 - (0.2753)^{\frac{3}{2}}]\,H^{\frac{3}{2}} = 2.5261\ \mu b H^{\frac{3}{2}}\ .\quad .\ (2)$$

This is so manifestly incorrect that scarcely any comment is necessary. Professor Ruehlmann remarks of this formula that it does not agree any better with experiments than do the Scheffler and Braschmann modifications of the Weisbach formula.

Lesbros experimented with very small overfalls (b = 8 inches; k = 20 inches; B = 12 feet) and computed the values μ from the fundamental formula

$$Q = \tfrac{2}{3}\ \mu b H \sqrt{2\,gH},$$

which applies only for discharge through a lateral orifice when there is no velocity of approach. He determined 2000 different values for μ, all for the constant condition $b/B = 0.054$. These coefficients are entirely without value and it is difficult to understand how any person could expend the mentality required for

these computations when the futility of the undertaking must have been apparent.

Weisbach later continued his experiments, commenced in 1842, and established the phenomena of incomplete contraction. On these experiments, made with 8-inch wide openings through the thin wall of a 14-inch wide flume, he based the two following formulæ:

When $b < B$

$$Q = \frac{2}{3}\mu\left[1 + 1.718\left(\frac{bH}{BT}\right)^4\right] bH\sqrt{2\,gH} \quad . \quad . \quad . \quad (3a)$$

and when $b = B$

$$Q = \frac{2}{3}\mu\left[1.041 + 0.3693\left(\frac{H}{T}\right)^2\right] bH\sqrt{2\,gH} \quad . \quad (3b)$$

The coefficients $\frac{2}{3}\ \mu$ are those given in Poucelet-Lesbros' tables based on 8-inch wide overfalls.

Here again the second factor represents the discharge through a lateral orifice and the first factor is a variable coefficient of irrational form. Besides the apparent incorrectness of the formula it is certain that coefficients derived from 8-inch openings are not applicable to Weisbach's experiments.

Boileau, in 1845, was induced to make other experiments on complete overfalls for cases where $b = B = 0.94$ to 5.31 feet. He gave the following formulæ for discharge and μ:

$$\left.\begin{array}{l} Q = \mu bH\sqrt{2\,gH}\,;\ \mu = \sqrt{\dfrac{1 - \dfrac{k}{H}}{1 - \dfrac{1}{\left(1 + \dfrac{k}{H}\right)^2}}} \\[3ex] \text{or} \qquad \mu = \dfrac{\sqrt{1 - \dfrac{e}{H}}}{\sqrt{1 - \left(\dfrac{1}{1 + \dfrac{k}{H}}\right)^2}} \end{array}\right\} \quad . \quad . \quad (4)$$

where $e = \overline{OE_1}$, Fig. 1.

It is clearly seen that Boileau did not consider the hydrodynamic pressure against the discharge area and the weir, nor the velocity of approach. Instead, his coefficient is made to cover the variations due to these pressures, the weir dimensions and contractions, etc. As previously shown, such formulæ cannot have any general applicability.

Redtenbacher, in 1848, proposed an empiric formula which was based on Castel's experiments on complete overfalls 0.4 to 29 inches wide. He gave

$$\left.\begin{aligned} Q &= \left(0.381 + 0.062\frac{b}{B}\right) bH\sqrt{2\,gH} \\ \text{and when } b = B,\ Q &= 0.443\, bH\sqrt{2\,gH}. \end{aligned}\right\} \quad (5)$$

According to Redtenbacher these formulæ are applicable only when $BT = 5\,bH$, and b is at least equal to $B/3$ and $k - T_1$ equals at least $2\,H$. Finally the weir must be sharp crested.

No consideration is given to hydrodynamic pressures due to the flow of approach, nor to the dimensions of the weir. The first term, which takes the place of μ, is a constant for all values of b/B = constant. It is hardly necessary to point out the uselessness of this formula.

J. B. Francis, in discussing his epoch-making "Lowell Hydraulic Experiments," 1855, modified Weisbach's formula to obtain the following:

$$Q = 3.33\,(b - 0.10\,nH)\,H_0^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

wherein n = number of end contractions;

H = measured height of water above the weir crest;

H_0 = a pressure height corrected for velocity of approach

and given by Francis as

$$H_0 = \left[\left(H + \frac{v^2}{2\,g}\right)^{\frac{3}{2}} - \left(\frac{v^2}{2\,g}\right)^{3}\right]^{\frac{2}{3}}.$$

In these experiments B = 13.96 feet; k = 2.04 feet to 5.05 feet; and b = 9.995 to 9.997 feet. The weir crest was the sharp plate since adopted as the standard form for experimental weirs.

The coefficient 3.33 in Eq. (6) is the average of values ranging between 3.3002 and 3.3617 and is a value for $\frac{2}{3} \mu \sqrt{2g}$. Hence $\mu = 0.6228$ is considered constant for all weir dimensions and depths of water, which is certainly wrong.

When there are no end contractions, $b = B$ and $n = 0$, and Eq. (6) becomes

$$Q = 3.33\, bH_0^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6b)$$

which cannot be regarded as a general law because when $b = 0.10\, nH$ in Eq. (6), then $Q = 0$, which is an apparent contradiction.

These objections, together with the assumption that the flow takes place over a height H_0 while in reality the height is only H, render the formula quite valueless except in special cases resembling the Francis experiments.

This in no wise vitiates the high value which the very accurate experiments of Mr. Francis possess, irrespective of any theoretic deductions which may now or have ever been drawn therefrom.

Braschmann, in 1861, proposed a formula based on the principle of least work. It was a modified form of Navier's formula using Castel's and Lesbros' experiments for the determination of his coefficients. The general form is

$$Q = \mu bH \sqrt{2\,gH} \text{ where } \mu = 0.3838 + 0.0368\,\frac{b}{B} + \frac{0.00053}{H} \quad . \quad (7)$$

The objections to this formula are apparent from the preceding.

Bornemann, in 1870, experimented with weirs for which $b = B = 3.8$ feet; $H = 2.75$ to 8.27 inches; and $\frac{H}{T} = 0.2$ to 0.8. His formula is,

$$\left.\begin{array}{l} \text{for } H < \frac{1}{3}T, \quad Q = \left(0.5673 - 0.1239\sqrt{\frac{H}{T}}\right) bH\sqrt{2\,gH}, \\ \text{for } H > \frac{1}{3}T, \quad Q = \left(0.6402 - 0.2862\sqrt{\frac{H}{T}}\right) b\,(H+z) \\ \qquad \sqrt{2\,g\left(H + \frac{v^2}{2\,g}\right)}. \end{array}\right\} \quad . \quad . \quad (8)$$

Bornemann himself points out that his formulæ are not applicable unless $b = B$, and expresses the hope that somebody may eventually succeed in deriving mathematically correct forms. The first formula does not include velocity of approach and the second formula does this by introducing the fictitious height $H + z$.

G. Hagen in his book "Die Stroeme," 1871, adopts the Eytelwein-Weisbach formula.

M. Becker, in 1873, gave the following formula:

For velocity of efflux:

$$V = \mu \sqrt{2\, g \left(\frac{4}{9} H + \frac{v^2}{2\, g} \right)}$$

and for quantity

$$Q = \mu b H \sqrt{2\, g \left(\frac{4}{9} H + \frac{v^2}{2\, g} \right)} \cdot \quad \Bigg\} \quad \ldots \quad (9)$$

This formula is based on the incorrect assumption that the velocity of approach exerts an hydrodynamic pressure against the discharge area only and that the mean velocity of efflux over the total depth corresponds to a pressure head $\frac{4}{9} H + \frac{v^2}{2\, g}$, which cannot be generally admitted.

K. Pestalozzi, gives the Eytelwein-Weisbach formula, which need not be repeated here.

Ruehlmann expresses the opinion that the scientific value of all the formulæ above given is very small. He believes, however, that they are applicable to cases closely resembling the experiments on which they were founded.

After presenting numerous examples he shows that, even within range of the experiments, the discharges found by formulæ 3, 4, 6, 7 and 8 differ by amounts varying from 12 to 19 per cent.

Bazin. The objections previously cited with reference to the Weisbach formula apply equally to the following Bazin formula, 1898, Ann. d. Ponts et. Ch., p. 223, where

$$Q = \mu b \sqrt{2\, g} \left[H + \frac{n v^2}{2\, g} \right]^{\frac{3}{2}} \cdot \quad \ldots \quad (10)$$

Cipolletti uses a modification of Weisbach's formula, also a simple form of the Francis formula, and, in a very unscientific manner, adapts these to some experiments of his own.

The attempt to use these experiments for the determination of μ in the foregoing chapter proved futile because the necessary weir measurements were not published in the report describing the experiments.

Fteley and Stearns, as a result of their very extensive hydraulic experiments, made in 1877 to 1879, published the following modification of the Francis formula:

$$Q = 3.31\, bH_0^{\frac{3}{2}} + 0.007\, b \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The addition of the last term alone distinguishes this from Eq. (6*b*) and hence the criticisms previously made to the Francis formula also apply here.

2. INCOMPLETE OVERFALLS.

Dubuat, under suppositions discussed in Chapter I, derived the following formulæ and gives $\mu = \mu_1 = 0.633$:

$$Q = \tfrac{2}{3}\, \mu bH_2 \sqrt{2\, gH_2} + \mu_1 bH_1 \sqrt{2\, gH_2} \quad . \quad . \quad (12)$$

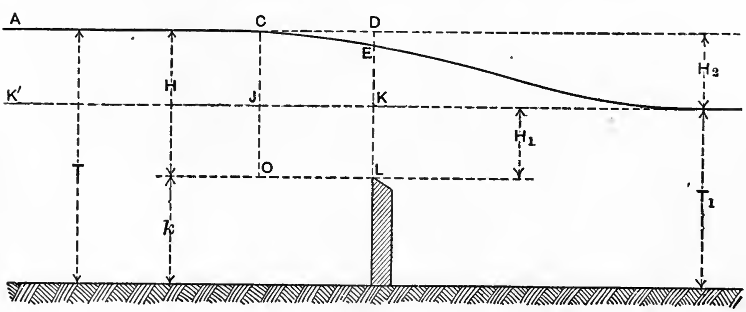


Fig. 2

Redtenbacher says that the derivation of formulæ for incomplete overfalls is connected with unsurmountable difficulties and adopts Dubuat's formula, making $\frac{2}{3}\,\mu = 0.57$ and $\mu_1 = 0.62$ both constant.

The first half of Eq. (12) is incorrect, because it is based on the assumption that discharge through the height H_2 takes place as for discharge into open air, which is not true. The second term does not include velocity of approach nor suction due to velocity of discharge. Ruehlmann advises against the use of these formulæ.

Lesbros based the following simple formula on his experiments made in 1829 to 1834.

$$Q = \mu b H \sqrt{2\,gH_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where μ is variable and made to depend on the ratio H_2/H. In principle, this formula is entirely wrong, assuming as it does that the discharge takes place over the whole height H and with a uniform velocity $\sqrt{2\,gH_2}$.

Bornemann, as a result of his experiments, made from 1866 to 1872, on overfalls 22 to 45 inches wide, gives this formula:

$$Q = b\sqrt{2g}\left[\frac{2}{3}\mu\left(H_2 + \frac{v^2}{2\,g}\right)^{\frac{3}{2}} - \left(\frac{v^2}{2\,g}\right)^{\frac{3}{2}}\right] + \mu_1 H_1\left(H_2 + \frac{v^2}{2\,g}\right)^{\frac{1}{2}}\Bigg] \quad (14)$$

in which $\mu = 0.702 - 0.2226\sqrt{\dfrac{H_2}{b}} + 0.1845\left(\dfrac{H_1}{H}\right)^2$

and $\dfrac{v^2}{2\,g} = \dfrac{1}{2\,g}\left(\dfrac{Q}{bT}\right)^2.$

The first member of Eq. (14) would be true provided the upper discharge did take place through the height H_2 into free air. The second term is incorrect because it assumes the submerged section as discharging into quiet water.

G. Hagen gives no formula for incomplete overfalls but states that the upper layer of flow may be regarded as a complete overfall and that the submerged portion is subjected to a uniform pressure, corresponding to the height H_1, thus leaving out of consideration the suction and assuming the counterpressure from the lower level active over the entire submerged area.

M. Becker proposes the formula

$$Q = \mu b H_2 \sqrt{2\,g\left(\frac{4}{9}\,H_2 + \frac{v^2}{2\,g}\right)} + \mu_1 b H_1 \sqrt{2\,g\left(H_2 + \frac{v^2}{2\,g}\right)} \quad (15)$$

The first term is based on the wrong assumption that the flow of approach exerts an hydrodynamic pressure on the discharge area only and that the mean discharge velocity corresponds to a pressure height $\left(\frac{4}{9} H_2 + \frac{v^2}{2\,g}\right)$. The second term neglects suction and assumes that the counterpressure is active over the whole submerged section.

K. Pestalozzi's formula, for which he gives $\mu = 0.8$ to 0.85 and $\mu_1 = 0.62$, is patterned after the Weisbach's formula for complete overfalls. It is

$$Q = \frac{2}{3}\,\mu b\sqrt{2\,g}\left[\left(H_2 + \frac{v^2}{2\,g}\right)^{\frac{3}{2}} - \left(\frac{v^2}{2\,g}\right)^{\frac{3}{2}}\right] + \mu_1 b H_1 \sqrt{2\,g\left(H_2 + \frac{v^2}{2\,g}\right)}. \tag{16}$$

The second term neglects suction effect and weir dimensions.

Messrs. Fteley and Stearns contributed a new formula for incomplete overfalls which is purely empiric and assumes both the upper and lower pools in a quiescent state. This formula is

$$Q = cb\left(H + \frac{H_1}{2}\right)\sqrt{H_2}$$

wherein c is a coefficient depending on the ratio H/H_1 and varies between 3.089 and 3.372.

While it is not supposed that any empiric formula will apply outside of the limits of the experiments for which it was proposed, it is not necessary, in view of what has gone before, to say more than that this formula has served its purpose well.

It was not until the experiments of Messrs. Francis, Fteley and Stearns had been made and given to the world that anyone had even the right to claim any great knowledge regarding the science of weir hydraulics. While there is still far more to do along the experimental line than the sum total work already accomplished, everyone must cherish a feeling of gratitude towards these gentlemen and express the fond hope that some day, some one else will continue this work.

Several large hydraulic laboratories have come into being during recent years and they should undoubtedly contribute something to our present knowledge which will furnish such data as are necessary for work of the kind here treated.

In concluding this subject it may be stated, that without exception, modern writers have adopted one or more of the formulæ given in this Appendix, sometimes without mentioning the source, hence this review is considered sufficiently exhaustive to show, without question, the irrational constitution of all older formulæ.

The attempt here made to offer something in a progressive direction would, therefore, seem justified. However, it is frankly admitted that the subject treated is still a very imperfect branch of engineering science.

APPENDIX B.

ON THE FLOW OVER A FLIGHT OF PANAMA CANAL LOCKS.

Solution of a Novel Hydraulic Problem.

In the design of a movable dam for the head of the triple flight of locks at Gatun, one of the first and most perplexing problems encountered was to determine the conditions of flow which would obtain in the event of serious accident to any of the upper lock gates.

Such a catastrophe, while highly improbable, is nevertheless not impossible, and in the remote case of its happening would cripple the operation of the entire Panama Canal until the resulting torrent pouring down through the locks could be effectively checked.

To meet this extraordinary contingency, it is proposed to erect a movable dam, probably of the swing bridge type. Omitting the essential details of the dam, it is sufficient to mention here that the strength of the structure and the power for its operation depend for their determination on the depth and velocity of flow through the section at which the dam is to be located.

For some time this flow problem seemed to offer unsurmountable difficulties owing to the many unknown quantities which necessarily enter. A thorough search through the world's hydraulic literature did not add much encouragement, and it became manifest that a solution, if one were possible, would mean a radical departure from any of the previously known methods for solving hydraulic problems. Also a solution, to be of any value in connection with the general question, must lay claim to considerable accuracy.

The triple flight of locks at Gatun presents a succession of

weirs or overfalls which may be complete or incomplete, depending on the profile of the locks and the total drop in the water levels between Gatun Lake and the Atlantic Ocean.

For the discharge through the straight portions of the locks and canal the Chezy formula with Bazin's coefficients will be used. This is probably the most reliable for flow through open channels having steep slope. For small slopes, the well tested formula of Ganguillet and Kutter may be more accurate.

Suppose now that acceptable formulæ are at hand for finding discharge over any single drop, also for the straight portions of the canal and locks. This then furnishes one formula for each condition of flow throughout the entire stretch of canal under discussion. Hence there are as many possible equations as there are varieties of conditions of flow.

Regarding the feasibility of solving any or all of these equations, it will be best to give the general forms and discuss them with relation to the known and unknown quantities.

For the straight portions of the channel and locks the Chezy formula gives

$$v = C\sqrt{rs} \text{ and } Q = Av = AC\sqrt{rs} \quad . \quad . \quad . \quad (1)$$

also $sL = H$, which, substituted into Eq. (1), gives

$$Q = AC\sqrt{\frac{rH}{L}}, \text{ wherein } C = \frac{87}{0.552 + \frac{m}{r}} \quad . \quad . \quad . \quad . \quad (2)$$

Q = quantity of discharge in cubic feet per second.
A = discharge section in square feet.
r = mean hydraulic radius in feet.
L = length of straight channel in feet.
H = fall in surface, in feet, over length L.
C = the Bazin Coefficient, wherein m is an experience number the values of which are given by Bazin for all conditions of flow. The values of m vary from 0.06 to 1.75, see under 2, Chapter VI.

For incomplete overfalls, the new formulæ, Eqs. (28), give

$$Q = b\sqrt{2\,g}\left[\frac{2}{3}\,\mu\,(S_1^{\frac{3}{2}} - S^{\frac{3}{2}}) + \mu_1\left(H_1 - \frac{nV^2}{2\,g}\right)\sqrt{\frac{S_1 + S_2}{2}}\right] \quad (3)$$

wherein $$S = \frac{v^2}{2\,g},\ S_1 = S + H_2 + \frac{nV^2}{2\,g}$$

and $$S_2 = S_1 + \frac{v^2 k}{g\left(H_1 - \frac{nV^2}{2\,g}\right)}.$$

Here b = uniform width of canal and overfall in feet.
g = acceleration of gravity = 32.16 feet.
v = mean velocity of approach in feet per second.
V = mean velocity of discharge in feet per second.
n = coefficient of contraction = 0.67.
k = height of weir above approach canal bottom.
H_1 = depth of weir crest below lower pool.
H_2 = depth of lower pool below upper pool.
T = depth of approach canal.
T_1 = depth of discharge section.

μ and μ_1 are discharge coefficients for free and submerged discharge respectively.

Then $$v = \frac{Q}{A} = \frac{Q}{bT};\quad V = \frac{Q}{bT_1};\ \text{and}\ H_1 = T - H_2. \quad . \quad . \quad (4)$$

The known quantities are b = 100 feet; T = 50 feet; and k, being small compared with the depth T, is neglected. All other quantities are unknown and depend for their values on Q and H_2. Hence, if Q and H_2 are regarded as the independent variables, the other quantities may be expressed in terms of these two. See Plate I, left hand end of lower profile, for lettered dimensions.

The upper profile of Plate I represents the condition prior to an accident, the lower profile gives the computed water surface down the flight of locks after a uniform condition of flow has been established.

While it is possible then to substitute numerical values into equations (3), involving only Q and H_2 as the final unknowns, it is quite impossible to solve directly the complicated form which results from such substitutions.

The approach velocity and quantity for the second drop is now represented by the discharge velocity and quantity from the first drop. Hence if the first case were solved the second could be solved in like manner, and so on for a third or fourth drop.

Now the quantity of discharge for a continuous flow must be constant, hence there is only one finally unknown, Q, while there is an unknown H_2 for each drop. This then enables writing out one equation for each H_2 in terms of Q, in which Q is a function of H_2 and itself. Thus:

$$\left.\begin{aligned} Q &= f\,(Q,\ H_2) \text{ for first drop;} \\ Q &= f^1\,(Q,\ H_2') \text{ for second drop;} \\ Q &= f''\,(Q,\ H_2'') \text{ for third drop;} \end{aligned}\right\} \quad \cdot\ \cdot\ \cdot\ \cdot \quad (5)$$

wherein there is one more unknown quantity than the number of equations. Hence the problem is not solvable until one other equation, involving these unknowns, is given.

In the same manner a series of equations may be written out for the horizontal stretches of the canal by using Eq. (2), thus:

$$Q = Ac\sqrt{\frac{rH'}{L}} = A'c'\sqrt{\frac{r'H''}{L'}} = A''c''\sqrt{\frac{r''H'''}{L''}}\text{, etc.,} \quad (6)$$

wherein all the quantities are known except Q, H', H'' and H''', and these equations also number one less than the number of unknowns.

However, the ocean level and the level for Gatun Lake being fixed, relatively, the total difference in their levels being 87 feet, the final condition follows:

$$\Sigma H_2 + \Sigma H' = 87 \quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \quad (7)$$

Hence putting Eqs. (5), (6) and (7) together this will give as many equations as there are unknowns, so the problem is definitely

solvable. But owing to the complexity of the equations there is no method known in algebra by which these equations can be solved for simultaneous values of the unknowns.

The solution given in the following is believed to be new and is original, as nothing bearing on this point could be found in any literature extant.

After much deliberation and study it was found that simultaneous values for the unknowns were obtainable by a graphic representation of Eq. (7), inasmuch as it is a straight line equation and depends for its fulfilment on a certain definite value of Q. Then if the values H_2 and H' are ascertained for all reasonable, assumed values of Q, and plotted as co-ordinates, there will result as many curves as there are equations less one.

The missing equation is Eq. (7) and, by trial, such a value of Q can be found for which Eq. (7) will be satisfied, and this furnishes the final solution.

The value of Q thus determined, all the particular values of H_2 and H' become known and the profile of the surface can be drawn. This also fixes the velocity for every section along the entire stretch of canal.

To exemplify the above reasoning, the complete solution will now be illustrated by reference to Plates I and II.

Since all of the equations (3) are so extremely involved and complicated that they are not directly solvable, it becomes necessary to assign values to Q and find values for H_2 by successive trials, continuing this process until the equation is satisfied. While this is a laborious operation it is far easier than at first appears and with a little experience an average computer can soon learn to solve a point by two or three approximations.

A rough idea as to the limits between which the unknown Q may be located, can be obtained by a preliminary inspection of the given conditions, and by observing that a maximum value for $V = 0.67 \sqrt{2\,gh}$. However, this may be twice as large as the real velocity.

In the present problem it was considered safe to assume that the first velocity of approach would have a value somewhere between 20 and 25 feet per second, although the theoretic v would be about 37 feet.

It was also reasonable to suppose that each overfall would be incomplete, as a careful inspection of the profile would lead one to foresee. Hence, the formula for incomplete overfalls was used.

But this assumption might have been erroneous, a fact which would be clearly indicated by the values H_2 resulting from a few preliminary computations. In the latter event the formula for complete overfalls would have to be employed.

Suppose then that we have chosen the appropriate Eq. (3) and that the required Q corresponds to some velocity between 20 and 25 feet. Also, assume values for v from 20 feet and up, one foot apart; this will give a sufficient number of points to plot a curve such as shown on Plate II, for the upper lock, drop 1.

From Eqs. (4), Q may be found for any assumed v when the section is known, and since the depth at the head of the canal must remain constant, the discharge section may be assumed constant at the entrance to the canal. Therefore, the assumed data, for which values of H_2 are sought, would be as follows:

Case (1)	$v =$ 20 ft.	$T =$ 50 ft.	$b =$ 100 ft.	$Q =$ 100,000 cu. ft.
Case (2)	21	50	100	105,000
Case (3)	22	50	100	110,000
Case (4)	23	50	100	110,000
Case (5)	24	50	100	115,000
Case (6)	25	50	100	120,000

The complete computation for the first point will now be given, and this will serve as an illustration for all of the computations. The problem is to solve Eq. (3) for the case when $Q = 100,000$ cubic feet per second, as per case (1). (See Plate I.)

The coefficient μ_1 and μ must first be found from Eq. (77*a*) by substituting proper values for H_2 and b.

H_2 is not known but since the total of the three drops is 87 feet, it is sufficiently close to assume $H_2 = 25$ feet, as the term in Eq. (77*a*) involving H_2 has very small influence on μ and does not enter into μ_1. Then with $b = 100$ feet, which is the constant width of the locks, Eqs. (77*a*) give

$$\frac{2}{3}\mu = 0.4001 + \frac{0.00799}{H_2} + 0.000146\, b = 0.4150 \text{ or } \mu = 0.5533,$$

$$\mu_1 = 0.5346 + 0.000146\, b = 0.5492.$$

From Table VIII it is seen that ρ would be less than unity, but as no experiments on submerged weirs were available and as the crest in our problem is really a somewhat narrow sill of the miter gates, it is on the safe side to assume $\rho = 1$.

Hence in the following computations these coefficients are used: $n = 0.67$, $\rho = 1$ and $\mu = \mu_1 = 0.55$.

(1) *Assume now that* $H_2 = 15$ *feet when* $Q = 100{,}000$ *cubic feet.* Then $v = \dfrac{100{,}000}{50 \times 100} = 20$ feet; $H_1 = T - H_2 = 35$ feet; and $T_1 = H_1 + 21 = 56$ feet, from which $V = \dfrac{Q}{bT_1} = 17.86$; $\dfrac{nV^2}{2\,g} = 3.32$ and $\mu b\sqrt{2\,g} = 441.375$. Also $S = \dfrac{v^2}{2\,g} = 6.20$ and $S_1 = S + H_2 + \dfrac{nV^2}{2\,g} = 6.20 + 15 + 3.32 = 24.52$. When $k = 0$, then $S_1 = S_2$ and $\sqrt{\dfrac{S_1 + S_2}{2}} = \sqrt{S_1}$, hence Eq. (3) becomes

$$Q = \mu b\sqrt{2\,g}\left[\frac{2}{3}\left(S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right) + \left(H_1 - \frac{nV^2}{2\,g}\right)\sqrt{S_1}\right] \quad . \quad (8)$$

Substituting all the above values into Eq. (8) and solving, then

$$Q = 441.375\left[\tfrac{2}{3}(121.4 - 15.4) + (35 - 3.32)\sqrt{24.52}\right] = 100{,}420\,. \quad (9)$$

(2) This indicates that the first assumption for H_2 was a little large and the operation is repeated for $H_2 = 14.75$ feet. It should

be noted that this change does not affect S and that a second computation is much easier. The new value gives

$$S_1 = S + H_2 + \frac{nV^2}{2g} = 24.24;\ H_1 = 35.25;\ \ T_1 = 56.25;\text{ and}$$

$V = 17.78$. Hence

$$Q = 441.375[\tfrac{2}{3}(119.4 - 15.4) + (35.25 - 3.288)\sqrt{24.24}] = 99989. (10)$$

The exact value of H_2 may now be found by interpolation between the values (9) and (10) by correcting the last value for 11 cubic feet, which gives $H_2 = 14.75$, because the correction would be in the fourth decimal. It should be mentioned here that interpolation is not permissible unless reasonably close values have been found on both sides of the true value.

In this manner the values H_2 in Table I, for *drop* 1, were found, and these when plotted gave the curve for *discharge for upper lock* at *drop* 1, see Plate II.

Now each one of the above assumptions for v and Q (which always fixes a definite value V, for velocity of discharge) furnishes the conditions for approach to the second drop, *drop* 2. This is very important and on this fact is based the simultaneous relation of Q with the several values H_2 subsequently found for each drop.

However, the discharge in passing over the length of the upper lock must have some slope sufficient to continue the discharge from the first drop. This slope and fall H' over a distance $L = 1000$ feet is now computed by Eq. (2) for each v above assumed, and these figures are given in Table I in the horizontal column *Upper Lock* (H') and plotted as curve AB, Plate II.

The various depths T_1, being the depths of discharge from the *first drop*, are now reduced by amounts H', giving new values $T' = T_1 - H'$, from which the new velocity of approach for the *second drop* is found for each of the previously assumed values of Q. Hence, by using the values T' in place of the former value

T, each case of Q may again be solved exactly as for the first drop. From Plate I the following values may be taken:

$$v' = \frac{Q}{100\ T'};\ H_1' = T' - H_2';\ T_1' = H_1' + 31 \text{ and } V' = \frac{Q}{100\ T_1'}.$$

Whence the same computations are repeated for the second drop and results entered in Table I and plotted on Plate II, as the *discharge curve for drop* 2.

By a repetition of this process to the third drop the new values become

$$v'' = \frac{Q}{100\ T''};\ H_1'' = T'' - H_2'';\ T_1'' = H_1'' + 29 \text{ and } V'' = \frac{Q}{100\ T_1''},$$

and after computing H_2'' for each assumed Q, the *discharge curve for drop* 3 was plotted on Plate II.

Finally, from Eqs. (2) or (6), the falls H' through the lower lock, the Approach Canal, and two miles of wide canal connecting with the ocean, may be found for each of the first assumed quantities of discharge, and a discharge curve may then be plotted for each channel.

Having thus found the related discharge curves for each condition of flow, over the entire stretch of canal and locks, the final solution is easily accomplished. Since for any value of Q, the accompanying values H_2 and H' are simultaneous values, made so by the previous method of computation, then such a value can be found, by trial, which will make $\Sigma H_2 + \Sigma H' = 87$ feet and the problem is solved.

Referring to Plate II, and the tabulation in Table I, the value $Q = 115{,}570$ cubic feet per second satisfies this final condition.

The figures in the last column of Table I were read from the curves of Plate II, excepting a few of the small drops which could better be interpolated from the table. These several values of H_2 and H' were then used to plot the surface slope for the entire stretch of canal, as shown in the lower profile of Plate I. From this profile the depth and velocity of flow at any discharge section

may be found by dividing the area of the section into 115,570. The upper approach velocity is thus found to be 23.2 feet.

Regarding the accuracy of the above method it is believed one-half per cent would cover all of the slight inaccuracies inherent in the solution of the equations, while the coefficients are to some extent speculative. It may be said then that the problem is solved with the greatest accuracy attainable with our present knowledge of the empiric coefficients, and these may be considerably in error when the differences in circumstances, for which they were determined, are considered.

Hence, we may conclude that such problems as the above are susceptible to general solution within the knowable accuracy of the empiric coefficients. This then emphasizes the importance of conducting larger hydraulic experiments especially for cases of deep and submerged flow.

TABLE I. — GIVING DROPS IN FEET FOR ASSUMED DISCHARGES.

Points for which Discharge was Found.		Discharge, Cubic Feet per Second.								Read off from Plate II.
		100,000	105,000	110,000	115,000	115,500	115,700	116,000	116,500	115,570
Approach	h	0.15	0.16	0.17	0.18	0.18	0.18	0.19	0.20	0.18
Drop 1	H_2	14.75	16.72	18.97	21.63	21.87	22.00	22.16	22.56	22.00
Upper lock	H'	0.63	0.79	0.95	1.20	1.24	1.25	1.27	1.30	1.24
Drop 2	H_2'	11.68	14.50	19.16	28.00	29.75	30.75	32.07	39.00	30.15
Middle lock	H''	0.31	0.39	0.54	0.88	0.96	0.98	1.04	1.11	0.97
Drop 3	H_2''	5.78	7.50	10.75	22.70	28.20	36.27	*	*	29.90
Lower lock	H^{III}	...	...	...	0.52	0.61	0.65	0.70	0.80	0.62
Canal 800 feet	H^{IV}	...	...	...	0.47	0.48	0.48	0.48	0.49	0.48
Canal 2 miles	H^{V}	...	...	...	1.45	1.46	1.47	1.48	1.49	1.46
Total drop =		$\Sigma H_2 +$	$\Sigma H' + h$	=	77.03	84.75	94.03	...	...	87.00 ft.

* When $H_1 < \frac{nV^2}{2g}$ in Eq. (3) the second term becomes negative and the condition may be regarded similar to a complete overfall although there is still some submerged flow.

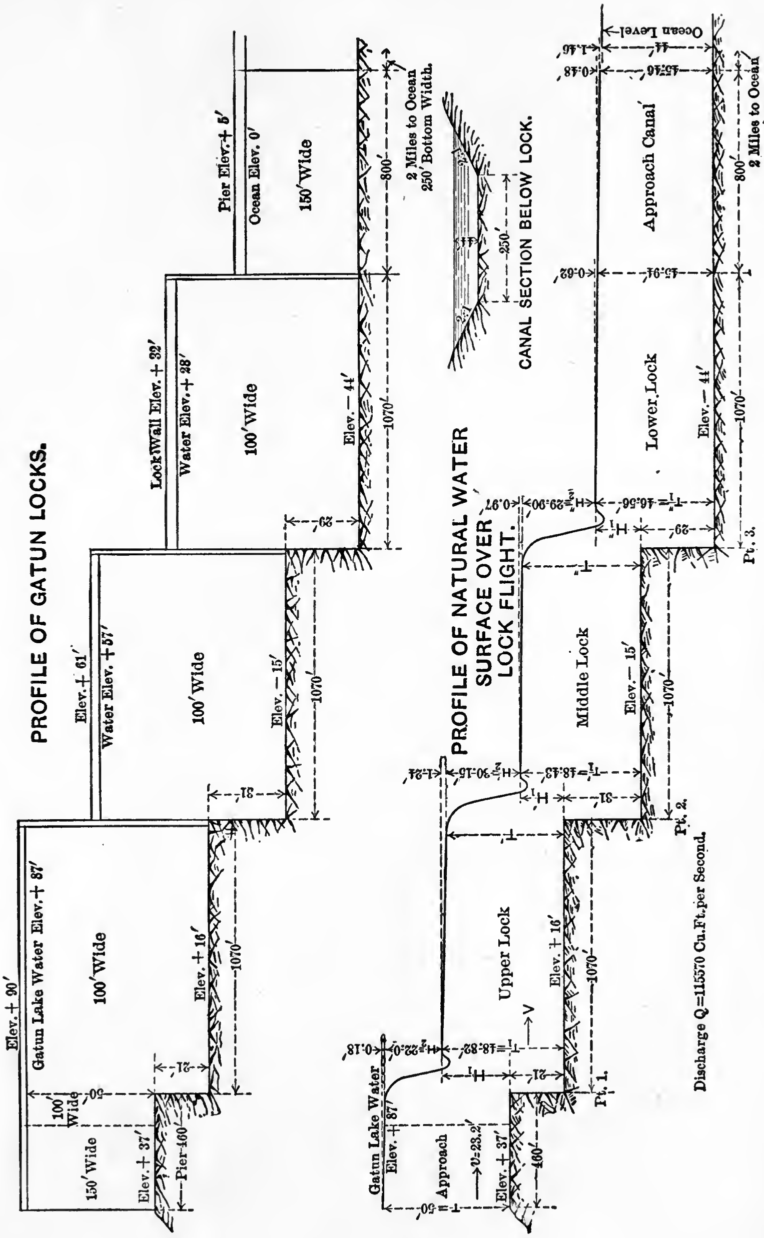
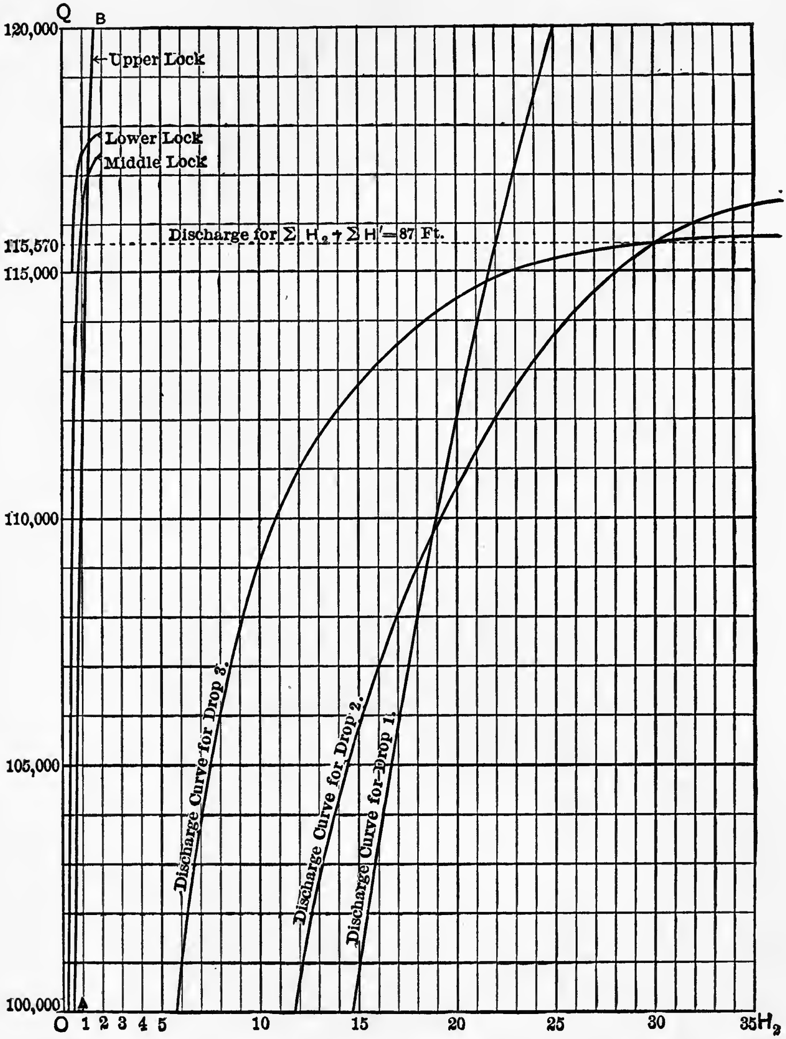

PROFILE OF GATUN LOCKS.
Elev. + 90′
Gatun Lake Water Elev. + 87′
100′ Wide
150′ Wide
Elev. + 37′
Pier 460′
Elev. + 16′
1070′
Elev. + 61′
Water Elev. + 57′
Elev. − 15′
Lock Wall Elev. + 32′
Water Elev. + 28′
Elev. − 44′
Pier Elev. + 5′
Ocean Elev. 0′
150′ Wide
800′
2 Miles to Ocean
250′ Bottom Width.
CANAL SECTION BELOW LOCK.
250′
PROFILE OF NATURAL WATER SURFACE OVER LOCK FLIGHT.
Gatun Lake Water Elev. + 87′
Approach
v=23.2
Upper Lock
Middle Lock
Lower Lock
Approach Canal
Ocean Level
Pt. 1.
Pt. 2.
Pt. 3.
2 Miles to Ocean
Discharge Q=115570 Cu.Ft.per Second.

PLATE I



DISCHARGE CURVES FOR GATUN LOCK FLIGHT. PANAMA CANAL.

PLATE II

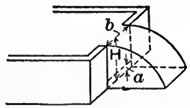
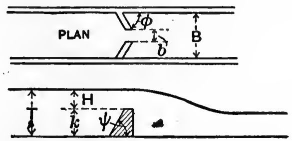
APPENDIX C.

A TABULATION OF THE NEW FORMULÆ.

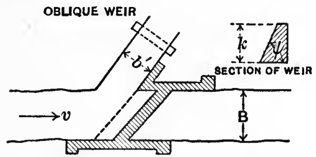
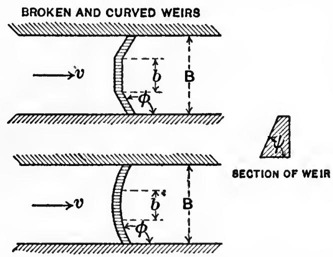
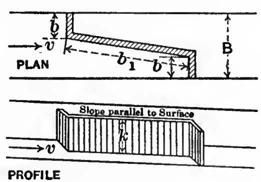
ARRANGED FOR READY REFERENCE.

NOTE—*For definitions of terms see p. IX.*

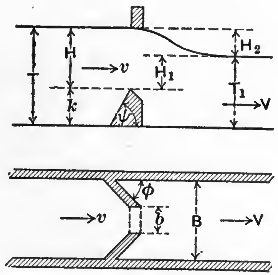
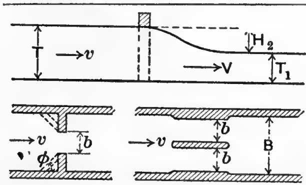
COMPLETE OVERFALL WEIRS.

Dimensions.	No. of Formula.	Formula for Discharge. Feet or Meters.	No. of Formula.	Formulæ for Coefficients for Sharp Crested Weirs where $\rho = 1$.
	1 2 3	$Q = \frac{2}{3}\mu bH\sqrt{2gH}$ for open orifice, $Q = \frac{2}{3}\mu b\sqrt{2g}\,(H^{\frac{3}{2}} - H_1^{\frac{3}{2}})$ for partly closed orifice, $Q = \frac{2}{3}\mu ab\sqrt{2g\left(H - \frac{a}{2}\right)}$ for small opening "a."		
	20 19	$S = \frac{v^2}{2g}\left[1 + \left(\frac{B-b}{b}\right)\cos^2\frac{\phi}{2}\right]$ where $v = \frac{Q}{B(k+H)}$, $S_1 = S + H + \frac{2Bk}{bgH}\,v^2\cos^2\frac{\psi}{2}$, $Q = \frac{2}{3}\mu b\sqrt{2g}\left(\frac{H}{S_1 - S}\right)[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}]$.	72 73	For Feet: $\frac{2}{3}\mu = 0.3655 + 0.02357\left(\frac{b}{B}\right) + \frac{0.007328}{H} + 0.00093\,b$ For Meters: $\frac{2}{3}\mu = 0.3655 + 0.02357\left(\frac{b}{B}\right) + \frac{0.002384}{H} + 0.00305\,b$.
$\phi = 90°$ and $\psi - 90°$	21	$S = \frac{v^2}{2g}\left[1 + \left(\frac{B-b}{2b}\right)\right]$ where $v = \frac{Q}{B(k+H)}$, $S_1 = S + H + \frac{Bk}{bgH}\,v^2$, $Q = \frac{2}{3}\mu b\sqrt{2g}\left(\frac{H}{S_1 - S}\right)[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}]$.	72 73	
No wingwalls; then $B = b$. Also $\phi = 90°$; $\psi = 90°$.	22	$S = \frac{v^2}{2g}$ where $v = \frac{Q}{b(k+H)}$, $S_1 = S + H + \frac{k}{gH}\,v^2$, $Q = \frac{2}{3}\mu b\sqrt{2g}\left(\frac{H}{S_1 - S}\right)[S_1^{\frac{3}{2}} - S^{\frac{3}{2}}]$.	70a 70b	For Feet: $\frac{2}{3}\mu = 0.3851 - 0.0258\left(\frac{H}{H+k}\right) + \frac{0.024}{H} + 0.000132\,b$. For Meters: $\frac{2}{3}\mu = 0.3851 - 0.0258\left(\frac{H}{H+k}\right) + \frac{0.00731}{H} + 0.00043\,b$.
$B = b$; $k = 0$; $\phi = 0$; $\psi = 0$.  This ceases to be a weir and becomes a vertical drop.	23	$S = \frac{v^2}{2g}$ where $v = \frac{Q}{bH}$, $S_1 = S + H = \frac{v^2}{2g} + H$, $Q = \frac{2}{3}\mu b\sqrt{2g}\left[\left(H + \frac{v^2}{2g}\right)^{\frac{3}{2}} - \left(\frac{v^2}{g}\right)^{\frac{3}{2}}\right]$ same as Weisbach's formula.	70a 70b	For small values of H use Eqs. (67) and (68).

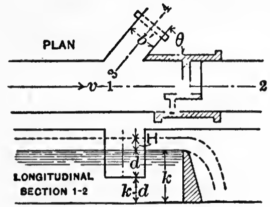
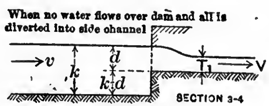
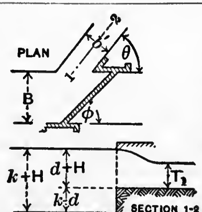
NOTE. — For values of ρ see Chapter VII under 6, Weir Crests.

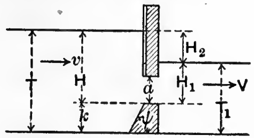
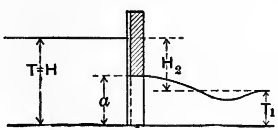
Dimensions.	No. of Formula.	Formula for Discharge. Feet and Meters.	No. of Formula.	Formulæ for Coefficients for Sharp Crested Weirs where $\rho = 1$.
	25	$S = \frac{v^2}{2g} \sin^2 \phi$ where $v = \frac{Q}{B(k+H)}$,	70a	NOTE: for small values of H use Eqs. (67) and (68).
		$S_1 = S + H + \frac{2k}{gH} v^2 \sin^2 \phi \cos^2 \frac{\psi}{2}$,		
		$Q = \frac{2}{3} \mu \frac{B}{\sin \phi} \sqrt{2g} \left(\frac{H}{S_1 - S}\right) [S_1^{\frac{3}{2}} - S^{\frac{3}{2}}]$,	70b	
	26	$S' = \frac{v^2}{2g}$ where $v = \frac{Q}{B(k+H)}$,		
		$S_1' = S' + H + \frac{2k}{gH} v^2 \cos^2 \frac{\psi}{2}$,	70a	
		$S = \frac{v^2}{2g} \sin^2 \phi$,		
		$S_1 = S + H + \frac{2k}{gH} v^2 \sin^2 \phi \cos^2 \frac{\psi}{2}$		
		$Q = Q_1 + Q_2 = \frac{2}{3} \mu H \sqrt{2g} \left[b \left(\frac{S_1'^{\frac{3}{2}} - S'^{\frac{3}{2}}}{S_1' - S'}\right) + \frac{B-b}{\sin \phi} \left(\frac{S_1^{\frac{3}{2}} - S^{\frac{3}{2}}}{S_1 - S}\right)\right]$,	70b	
	27	$S = \frac{v^2}{2g}$ where $v = \frac{Q}{B(k+H)}$,	70a	
		$S_1 = S + H + \frac{k}{gH} v^2$,		
		$Q = 2q + q_1 = \frac{2}{3} \mu H \sqrt{2g} \left[2b \left(\frac{S_1^{\frac{3}{2}} - S^{\frac{3}{2}}}{S_1 - S}\right) + b_1 H^{\frac{1}{2}} \right]$.	70b	

NOTE. — For values of ρ see Chapter VII, under 6, Weir Crests.

Dimensions.	No. of Formula.	Formula for Discharge. Feet and Meters.	No. of Formula.	Formulæ for Coefficients μ and μ_1 for Sharp Crested Weirs, where $\rho = 1$.
	28	$S = \frac{v^2}{2g}\left[1 + \left(\frac{B-b}{b}\right)\cos^2\frac{\phi}{2}\right],$ $S_1 = S + H_2 + \frac{nV^2}{2g},$ $S_2 = S_1 + \frac{2v^2\ Bk\cos^2\frac{\psi}{2}}{bg\left(H_1 - \frac{nV^2}{2g}\right)},$ $Q = Q_1 + Q_2 =$ $\frac{2}{3}b\sqrt{2g}\left[\mu\left(S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right) + \mu_1\left(H_1 - \frac{nV^2}{2g}\right)\left(\frac{S_2^{\frac{3}{2}} - S_1^{\frac{3}{2}}}{S_2 - S_1}\right)\right].$ When H_1 is small: $Q = Q_1 + Q_2 =$ $b\sqrt{2g}\left[\frac{2}{3}\mu\left(S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right) + \mu_1\left(H_1 - \frac{nV^2}{2g}\right)\sqrt{\frac{S_1 + S_2}{2}}\right].$	77a 77b	For larger values of H_2. For feet $\frac{2}{3}\mu = 0.4001 + \frac{0.00799}{H_2} + 0.000146\,b,$ $\mu_1 = 0.5346 + 0.000146\,b.$ For meters $\frac{2}{3}\mu = 0.4001 + \frac{0.00244}{H_2} + 0.00048\,b,$ $\mu = 0.5346 + 0.00048\,b.$ For small values of H_2 use formula 76. $n = 0.67.$ Ditto.
Same as above when $v = 0$.	29a	$S = 0;\ S_1 = S_2 = H_2 + \frac{nV^2}{2g},$ $Q = Q_1 + Q_2 = b\sqrt{2g\left(H_2 + \frac{nV^2}{2g}\right)}\left[\frac{2}{3}\mu\left(H_2 + \frac{nV^2}{2g}\right)\right.$ $\left. + \mu_1\left(H_1 - \frac{nV^2}{2g}\right)\right],$		Ditto.
Same as above when $v = V = 0$; $\psi = 90°$ and $\phi = 90°$. Becomes same as Dubuat's form.	29b	$S = 0;\ S_1 = S_2 = H_2,$ $Q = Q_1 + Q_2 = b\sqrt{2gH_2}\left(\frac{2}{3}\mu H_2 + \mu_1 H_1\right).$		Ditto.
	30	$S = \frac{v^2}{2g}\left[1 + \left(\frac{B-b}{b}\right)\cos^2\frac{\phi}{2}\right],$ $S_1 = S + H_2 + \frac{nV^2}{2g} = S_2,$ $Q = Q_1 + Q_2 =$ $b\sqrt{2g}\left[\frac{2}{3}\mu\left(S_1^{\frac{3}{2}} - S^{\frac{3}{2}}\right) + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\sqrt{S_1}\right].$		Ditto. NOTE: There are no experiments from which to determine ρ for incomplete overfalls.

NOTE.—When $H_1 = \frac{nV^2}{2g}$ in eqs. (28) and (29) then $S_2 = \infty$ and the overfall becomes complete. The last term in the equations for Q cannot have a real negative value.

Dimensions.	No. of Formula.	Formula for Diverted Discharge. Feet or Meters.	No. of Formula.	Formulæ for Coefficients μ.
	59	$S = \frac{v^2}{g}(0.25 + \sin^2\theta)$, $S_1 = S + d - \left(T_1 - \frac{nV^2}{2g}\right)$, $S_2 = S_1 + \frac{2v(k-d)}{g\left(T_1 - \frac{nV^2}{2g}\right)} \sin^2\theta \cos^2\frac{\psi}{2}$, $Q = Q_1 + Q_2 =$ $\frac{2}{3} b' \sqrt{2g}\left[\mu (S_1^{\frac{3}{2}} - S^{\frac{3}{2}}) + \mu_1 \left(T_1 - \frac{nV^2}{2g}\right)\left(\frac{S_2^{\frac{3}{2}} - S_1^{\frac{3}{2}}}{S_2 - S_1}\right)\right]$.		In the absence of all experiments on cases of this kind use Formulæ 77 with a suitable correction for wide crest.
	60	For $v = 0$. $S = 0;\ S_1 = S_2 = d - T_1 + \frac{nV^2}{2g}$, $Q = Q_1 + Q_2 = b'\sqrt{2g\left(d - T_1 + \frac{nV^2}{2g}\right)}$ $\left[\frac{2}{3}\mu\left(d - T_1 + \frac{nV^2}{2g}\right) + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\right]$.		Ditto.
When part of river flow, depth H, passes over dam-weir and depth $d + H$ passes out of side channel. Q = discharge through side channel. These formulæ are necessarily approximate, because of the many cross-currents in front of the discharge area.	61 and 62 a,b,c,d	$S_1 = d + H + \frac{nV^2}{2g} - T_1$, $S_2 = S_1 + \frac{v^2}{g}\left(\frac{k-d}{T_1 - \frac{nV^2}{2g}}\right)\sin\theta\cos^2\frac{\psi}{2}$, $Q = Q_1 + Q_2 =$ $\frac{2}{3} b'\sqrt{2g}\left[\mu S_1^{\frac{3}{2}} + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\frac{S_2^{\frac{3}{2}} - S_1^{\frac{3}{2}}}{S_2 - S_1}\right)\right]$.		Ditto.
Same as formulæ 61 and 62 when v and $k - d$ are small. Q = discharge through side channel.	62e	$S_2 = S_1 = d + H + \frac{nV^2}{2g} - T_1$, $Q = Q_1 + Q_2 = b'\sqrt{2gS_1}\left[\frac{2}{3}\mu S_1 + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\right]$.		Ditto.
 Part of flow diverted and part discharged over dam-weir as above.	65	$S = \frac{v^2\sin\phi}{g\sin Q}\cos(\theta - \phi) + \left[d + H + \frac{nV^2}{2g} - T_1\right]$, $S_1 = S + \frac{2v^2}{g}\left[\frac{k-d}{T_1 - \frac{nV^2}{2g}}\right]\left(\frac{\sin\phi}{\sin Q}\right)\cos^2\frac{\psi}{2}$ $Q = Q_1 + Q_2 = \frac{2}{3} b'\sqrt{2g}\left[\mu\left(d + H + \frac{nV^2}{2g} - T_1\right)\sqrt{S}\right.$ $\left. + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\left(\frac{S_1^{\frac{3}{2}} - S^{\frac{3}{2}}}{S_1 - S}\right)\right]$.		Ditto.

Dimensions.	No. of Formula.	Formula for Discharge. Feet or Meters.	No. of Formula.	Formulæ for Coefficients μ. To be Used with Reserve. All for Dimensions in Feet.
	36 a,b,c	$S = \frac{v^2}{2g}\left[1 + \frac{B-b}{2b} + \frac{B}{2ab}(T_1 + H_2 - (k+a))\right] + H_2 + \frac{nV^2}{2g}$, $S_1 = S + \frac{2v^2Bk}{abg}\cos^2\frac{\psi}{2}$, $Q = \frac{2}{3}\mu_1 ab\sqrt{2g}\left[\frac{S_1^{\frac{3}{2}} - S^{\frac{3}{2}}}{S_1 - S}\right]$,	79a 78a	For small sluices: $\mu_1 = 0.4988 + \frac{0.271\sqrt{a}}{H - \frac{a}{2}} + 0.00093\,b.$ For large regulating gates: $\mu_1 = 0.7069 - \frac{0.2717\sqrt{a}}{H - \frac{a}{2}} + 0.00093\,b.$ Both without bottom contraction. $n = 0.67.$
Same as above when $k = 0$.	36d	$S = S_1 = \frac{v^2}{2g}\left[1 + \frac{B-b}{2b} + \frac{B}{2ab}(T_1 + H_2 - a)\right] + H_2 + \frac{nV^2}{2g}$, $Q = \mu_1 ab\sqrt{2gS}$,		Ditto.
Same as above when $v = 0$ and $k = 0$.	39	$S = S_1 = H_2 + \frac{nV^2}{2g}$, $Q = \mu_1 ab\sqrt{2gS}$		Ditto.
	38	$S = \frac{v^2}{2g}\left[1 + \frac{B-b}{2b}\right] + (H_2 + T_1 - a)\left[1 + \frac{Bv^2}{4abg}\right]$, $S_1 = S + a + \frac{nV^2}{2g} - T_1$, $Q = Q_1 + Q_2$ $= b\sqrt{2g}\left[\frac{2}{3}\mu(S_1^{\frac{3}{2}} - S^{\frac{3}{2}}) + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\sqrt{S_1}\right]$.	80 81 82	For free discharge with complete contraction. $\mu = 0.5708 + \frac{0.01355\sqrt{a}}{\sqrt{H_2 + \frac{a}{2}}} + \frac{0.0382}{\sqrt{a}} + 0.00131\,b.$ For free discharge when $B = b$, without contraction. $\mu = 0.5751 - \frac{0.01898\sqrt{a}}{\sqrt{H_2 - \frac{a}{2}}} + \frac{0.00492}{a} + 0.000146\,b.$ For same as previous case with bottom edge of gate rounded. $\mu = 0.8452 - \frac{0.21936\sqrt{a}}{\sqrt{H_2 - \frac{a}{2}}} + \frac{0.00718}{a} + 0.000146\,b.$
Same as above when $v = 0$.	40	$S = H_2 + T_1 - a$, $S_1 = H_2 + \frac{nV^2}{2g}$, $Q = Q_1 + Q_2$ $= b\sqrt{2g}\left[\frac{2}{3}\mu(S_1^{\frac{3}{2}} - S^{\frac{3}{2}}) + \mu_1\left(T_1 - \frac{nV^2}{2g}\right)\sqrt{S_1}\right]$.		Ditto. See text for corresponding formulæ for metric units.

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